ACCELERATED MATHEMATICS

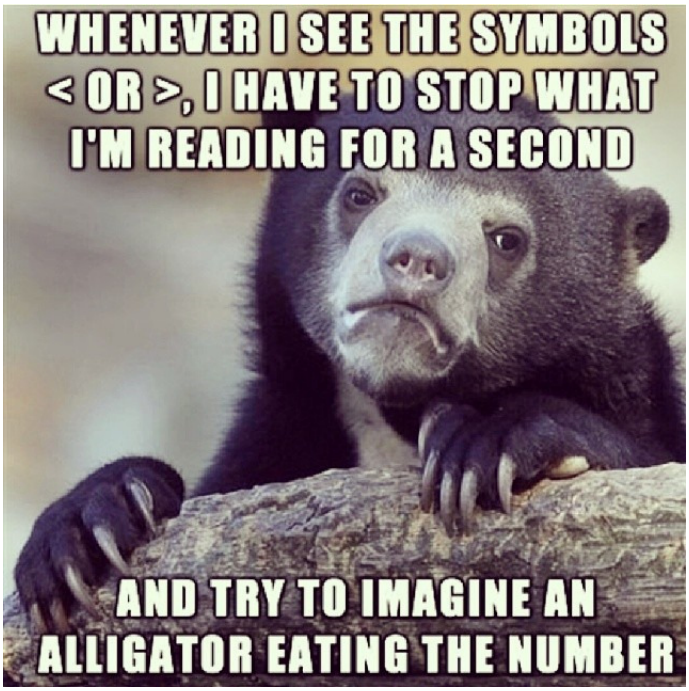
CHAPTER 4

PART II

INEQUALITIES

TOPICS COVERED:

- Solving inequalities with adding and subtracting
- Solving inequalities with multiplying and dividing
- Solving two-step inequalities
- Solving inequality word problems



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Activity 4II-1: **Describing the Ideal School with Inequalities** Name:

(Taken from *The Language of Algebra*)

These variables represent information about a particular school.

t = number of teachers at the school	m = number of math teachers at the school
b = the number of boys at the school	g = the number of girls at the school
p = number of class periods in one day	l = length of one class period, in minutes

1.	Add six more variables to the list above.

Using the variables above write at least five inequalities that describe the ideal school. Keep a record of what the inequalities represent.

Ex.	$b + g \leq 1{,}500$	There are less than or equal to 1,500 students.
2.		
3.		
4.		
5.		
6.		

Suppose that g represents the number of girls in a particular class and b represents the number of boys. Write each inequality using symbols. Then give the range for the number of girls if the number of boys is 20.

7.	The number of girls is less than or equal to half the number of boys.		
8.	The number of girls is less than 2 more than the number of boys.		
9.	The number of boys is at least 3 times the number of girls.		

Write three inequalities of your own using variables g and b. Write each one using symbols and words. Use each of the four operations at least once.

10.		
11.		
12.		

The prefix "in" means not, therefore an inequality is something that is not equal. The following symbols are often used with inequalities:

$>$	greater than	$<$	less than
$\geq$	greater than or equal to	$\leq$	less than or equal to
$\neq$	not equal		

An inequality statement can be read from left to right or from right to left. We usually use the variable as our starting point.

Example: $x < 5$ can be read "x is less than 5" or "5 is greater than x".
Example: $-3 \geq y$ can be read "y is less than or equal to -3" or "-3 is greater than or equal to y".

Which inequality symbols would be used for word problems with the following?

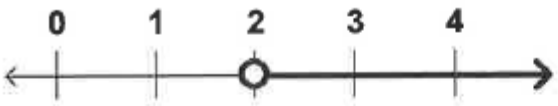
A. At most	E. Less than
B. At least	F. No less than
C. Cannot exceed	G. More than
D. Must exceed	H. No more than

State whether each inequality is true or false for the given value. Show all work.

1.	$b+10<12,\ b=4$	2.	$3<x-8,\ \mathrm{x}=12$
3.	$6m+3\leq 8,\ \mathrm{m}=1$	4.	$12\leq 2p-6,\ \mathrm{p}=9$
5.	$k-12<18,\ \mathrm{k}=31$	6.	$13>4+c,\ \mathrm{c}=9$
7.	$15+n\geq 15,\ \mathrm{n}=6$	8.	$2\geq t-3,\ \mathrm{t}=3$
9.	$4t-4<20,\ \mathrm{t}=7$	10.	$29<24+a,\ \mathrm{a}=6$
11.	$10\geq 2a+4,\ a=4$	12.	$5v>25,\ \mathrm{v}=4$
13.	$21<\frac{r}{3},\ \mathrm{r}=66$	14.	$\frac{s}{8}\geq 4,\ \mathrm{s}=32$
15.	$5w+8\leq 12,\ \mathrm{w}=0$	16.	$2y-7<41,\ \mathrm{y}=16$
17.	$3z+z-6<11,\ z=4$	18.	$5f-2f+3\geq 9,\ \mathrm{f}=2$
19.	$6h-3>15,\ \mathrm{h}=2$	20.	$81+3d\geq 90,\ \mathrm{d}=2$

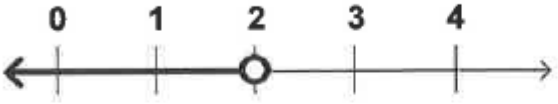
Grade 7 Equations & Inequalities on Number Line Foldable

Equations & Inequalities Solution Representations on Number Lines



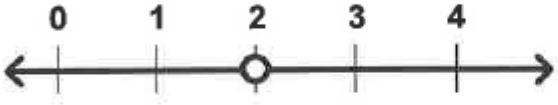
***x* is greater than 2; $x > 2$**

Greater Than



***x* is less than 2; $x < 2$**

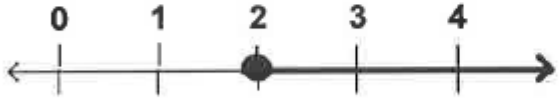
Less Than



***x* is not equal to 2; $x \neq 2$**

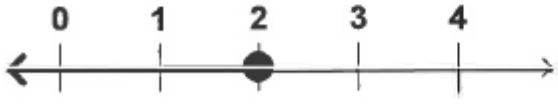
Not Equal to

Open Circle



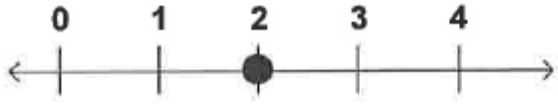
***x* is greater than 2; $x \geq 2$**

Greater Than or Equal to



***x* is less than 2; $x \leq 2$**

Less Than or Equal to



***x* is equal to 2; $x = 2$**

Equal to

Closed Circle

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Activity 4II-3: **Inequalities** Name:

Evaluate each expression if a=2, b=4, and c=6. Then write >, <, or = to make each sentence true.

1.	bc ac	2.	$c+6$ $3a+2c$
3.	$5b-2a$ $4b$	4.	$3c$ $2b+4a+2$
5.	$4c-5b$ $b-a$	6.	$5c-3b-a+16$ 0

Write an inequality for each sentence.

7.	Applicants with less than 5 years' experience must take a test.	
8.	The home team needs more than 6 points to win.	
9.	The minimum voting age is 18.	
10.	You must answer at least 10 questions correctly to stay in the game.	
11.	A tip of no less than 10% is considered appropriate.	
12.	The cost including tax is no more than $75.	
13.	The maximum load for an elevator is 2900 pounds.	
14.	A car can seat up to 8 passengers.	
15.	No persons under the age of 18 are permitted.	
16.	You must be at least 13 years old to join.	
17.	You should not drive over the speed limit of 70 mi/h.	
18.	To get a driver's license, a driver, d, should be at least 16 years old.	
19.	The forecasted temperature, t, will not exceed 82 degrees.	

>	**Greater Than**
≥	**Greater Than or Equal To** (The line underneath the greater than sign indicates also equal to)
<	**Less Than** (Tip: To remember this sign, if you open the sign up a little more, it would look like a capital L for less than)
≤	**Less Than or Equal To** (The line underneath the less than sign indicates also equal to)

Graphing Symbols

Greater Than (The open circle indicates that this is **NOT Equal to** the numeral graphed.

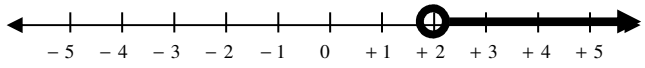
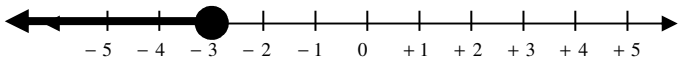
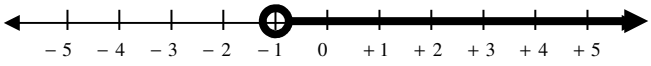
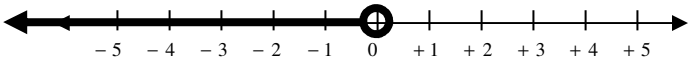
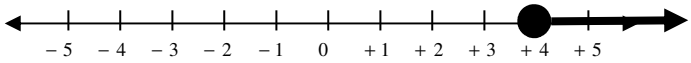
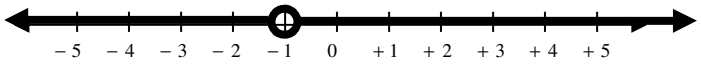
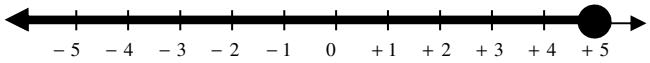
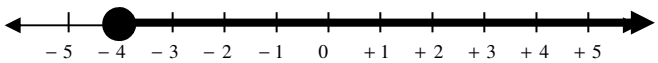
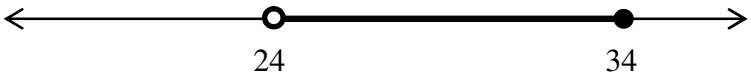
Greater Than or Equal To (The closed circle indicates that this is **Equal to** the numeral graphed.

Less Than (The open circle indicates that this is **NOT Equal to** the numeral graphed.

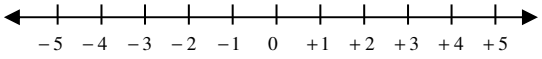
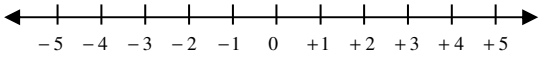
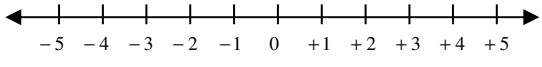
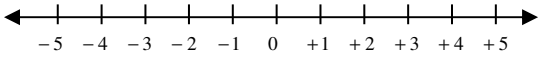
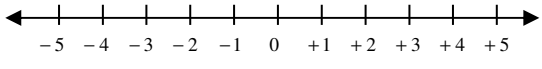
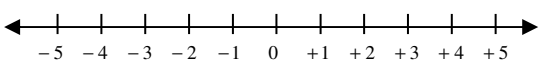
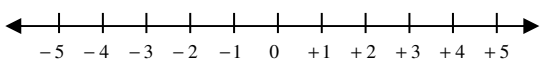
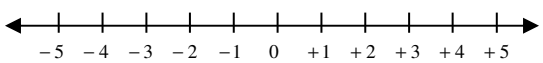
Less Than or Equal To (The closed circle indicates that this is **Equal to** the numeral graphed.

Activity 4II-4: **Addition and Subtraction with Inequalities** Name:

Write an inequality for each solution set graphed below.

1.		
2.		
3.		
4.		
5.		
6.		
7.		
8.		
9.	Interpret the graph. This graph shows the range of temperature in degrees Fahrenheit during a day in February.  24 34	

Solve each inequality showing all work on a separate sheet of paper. Check your solution. Then graph the solution on the number line.

10.	$x+3\geq 1$ 	11.	$x-8>-6$ 
12.	$x+21>25$ 	13.	$-12+x\leq -16$ 
14.	$-3>x-4$ 	15.	$x+1\frac{1}{2}>2\frac{1}{2}$ 
16.	$x-7\geq -11$ 	17.	$x-6>-6$ 

The Golden Rule of Inequalities

Whenever you multiply or divide both sides of an inequality by a ***negative number***, you must flip the inequality symbol.

1.	Isolate the variable by solving the inequality.
2.	Check the order. Variable on left side if preferred.
3.	Circle the number on the number line. Open or closed circle?
4.	Shade appropriately.

Open Circle

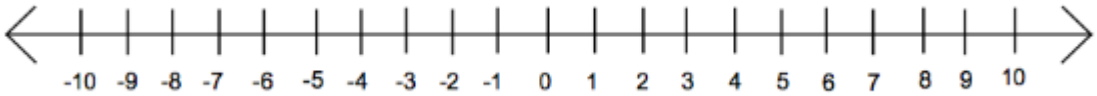
$<, >, \neq$

Closed Circle

$\leq, \geq, =$

Example: Solve and Graph

$5 - 3x \leq 13 + x$



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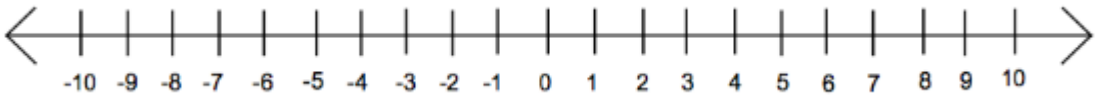

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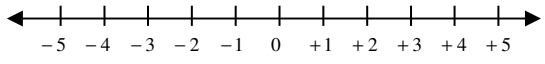
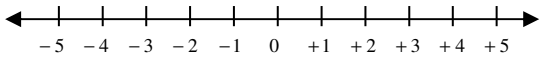
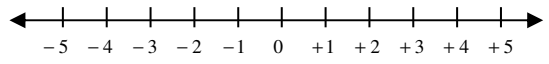
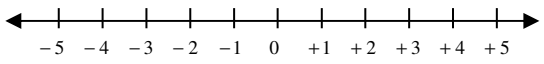
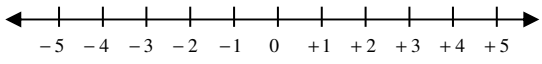
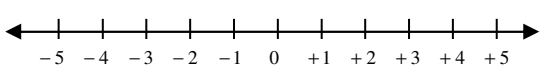
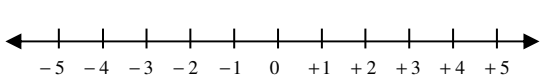
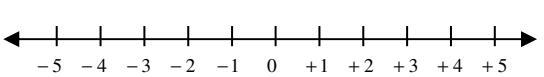
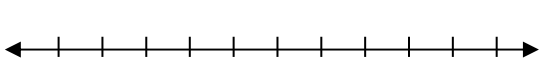
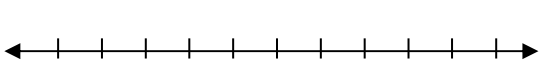
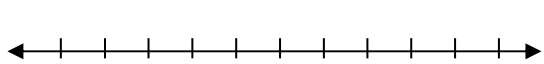
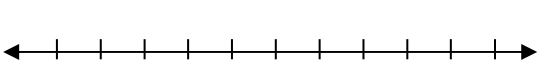
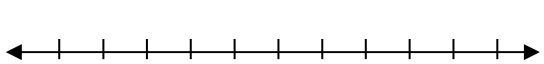
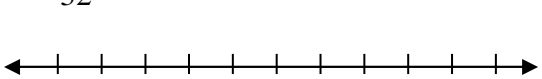
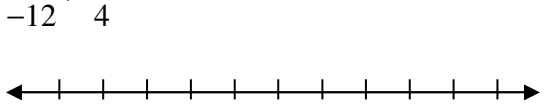
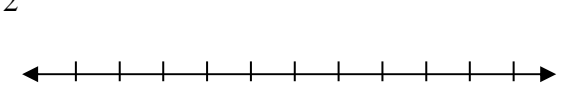
Activity 4II-5: **Multiplying and Dividing with Inequalities** Name:

Solve each inequality showing all work on a separate sheet of paper. Check your solution. Then graph the solution on the number line.

1.	$-5x < -25$ 	2.	$4x \geq -8$ 
3.	$\frac{b}{2} > 2$ 	4.	$\frac{x}{18} < \frac{1}{18}$ 
5.	$3x \geq 3$ 	6.	$-2x < -4$ 
7.	$\frac{c}{3} \leq -1$ 	8.	$-6x < 0$ 
9.	$-4x \geq 16$ 	10.	$\frac{w}{-1} \geq -5$ 
11.	$\frac{1}{-4} < \frac{m}{-4}$ 	12.	$2 \leq \frac{t}{-1}$ 
13.	$3x > -6$ 	14.	$\frac{-1}{8} \leq \frac{n}{-32}$ 
15.	$\frac{x}{-12} > \frac{1}{4}$ 	16.	$\frac{-1}{2}x \leq 2$ 

Take the following example:

$-3x + 5 > 17$

$+3x \quad +3x$

$5 > 17 + 3x$

$-17 \quad -17$

$\frac{-12}{3} > \frac{3x}{3}$ From this we see that x must be less than -4.

$-4 > x$

Therefore when we solve it the fastest way:

$-3x + 5 > 17$

$-5 \quad -5$

$\frac{-3x}{-3} > \frac{12}{-3}$

$x < -4$ The sign must switch in order to match our answer above.

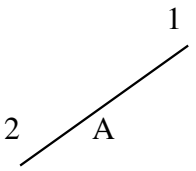
Another way to think about it is as a balanced scale

$2 = 2 \qquad \underline{2 \qquad 2}$

A

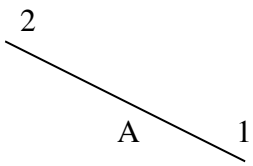
an inequality can be thought of as an unbalanced scale:

$2 > 1$



Multiplying it by -1 is like reversing gravity, so that things fall up, and the heavier object is now pulled more strongly up rather than down. If you do that, the scale will reverse:

$-2 < -1$

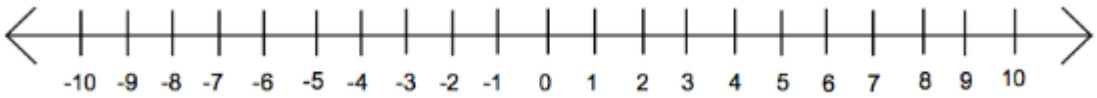
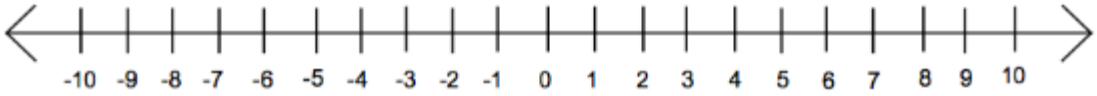
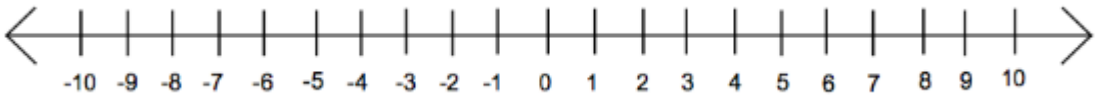
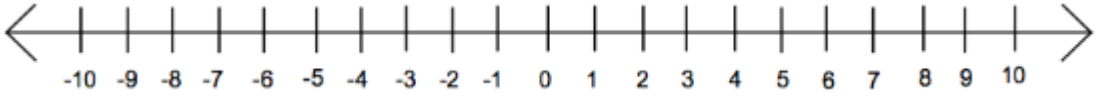


Activity 4II-7: **Solving Inequalities & Equations** Name:

Solve each inequality showing all work on a separate sheet of paper. Check your solution.

1.	$-8a < -5.68$		2.	$\frac{b}{4.2} \geq -5$	
3.	$27.44 > -4.9c$		4.	$-\frac{d}{4} < 4.7$	
5.	$-12.6 \leq 3n$		6.	$-5f > 35$	
7.	$\frac{k}{1.5} < -40$		8.	$20.4 \leq -3.4d$	
9.	$-1.27y \geq 0.0381$		10.	$-\frac{z}{2.1} < -100$	
11.	$-0.16s > -9.6$		12.	$-\frac{2}{3}t \leq -\frac{8}{9}$	
13.	$3.4j > 0.816$		14.	$\frac{r}{7.4} \geq -0.5$	

Write an inequality for each problem and solve showing all work on a separate sheet of paper.

15.	A road has a speed limit of 30 mph. Write an inequality that describes the legal speeds for motor vehicles.	
16.	Write an inequality for the sentence: c is not less than zero. Graph the solution on a number line. 	
17.	Solve the inequality: $a+4 \geq 8$ Graph the solution. 	
18.	Solve the inequality: $q-7 > 0$ Graph the solution. 	
19.	Solve the inequality: $h-2 < -1$ Graph the solution. 	

Solving Two-Step Inequalities

Example: Solve $-3y-12>-78$.

We are solving for *y* so we need to get *y* on one side of the equation all by itself. We must undo both the multiplication and the subtraction. Since we are undoing each operation, we work backwards through the order of operations: add first, then divide.

$$\begin{aligned} -3y-12 &> -78 \\ +12 \quad &\quad +12 \\ \frac{-3y}{-3} &> \frac{-66}{-3} \\ y &< 22 \end{aligned}$$

Solve each inequality showing all work on a separate sheet of paper. Check your solution.

1.	$2a+5<13$	2.	$7+6y\geq 55$	3.	$6t-1-8t\leq 15$
4.	$5x-9>21$	5.	$-10g+15\leq 95$	6.	$-\frac{n}{2}-12\leq -20$
7.	$9-3z<72$	8.	$3x+5+x>-27$	9.	$\frac{a}{3}+7>11$
10.	$5(w-10)\geq 105$	11.	$-2(k+6)<38$	12.	$5g-(g+1)>19$

Define a variable and then write an inequality for each problem and solve showing all work on a separate sheet of paper.

13.	Wade wants to buy two shirts and a tie and must spend no more than $60.00. The tie he likes costs $12.50. If the two shirts are the same price, how much can he pay for each shirt?
14.	In a new school auditorium, folding chairs 16 in. wide are to be installed side by side to form a row that is no longer than 25 ft. How many chairs will fit into this row?
15.	Patio blocks are packaged in bundles containing enough blocks to cover 16 ft.^{2} Sophie wants to build a patio having dimensions 18 ft by 13 ft. What is the least number of bundles of blocks she should order?
16.	An integer *k* divided by -3 is greater than 1. What is the greatest possible value of the integer?
17.	The sum of three consecutive even integers is less than -25. Find the 3 largest possible integers.
18.	Mr. Mangham started with $300 in his savings account. Each month he has to pay for his voice lesson, which is a $40 monthly fee. With *x* equal to the number of months of lessons, write an equation to represent the balance in Mr. Mangham's savings account each month.
19.	In the problem above, the bank will close his account if the balance drops below $50. To keep the account open, the balance must be greater than or equal to $50. Write an inequality to represent this situation.

1. Solve $0.5x - 2 < 5.5$ and graph the solution on a number line.

4. Solve $2.2x - 5.85 < -13$ and graph the solution on a number line.

5. Solve $0.4x - 4 < 2.4$ and graph the solution on a number line.

6.	Mrs. Smith wrote "Eight less than three times a number is greater than fifteen" on the board. If x represents the number, which inequality is a correct translation of this statement? $3x - 8 > 15$ $3x - 8 < 15$ $8 - 3x > 15$ $8 - 3x < 15$	
7.	The sign shown below is posted in front of a rollercoaster ride at Six Flags. If *h* represents the height of a rider in inches, what is the correct translation of the statement on this sign? 	

Solve and graph each inequality. Select one value from the shaded portion of the number line and verify that it is a solution to the inequality.

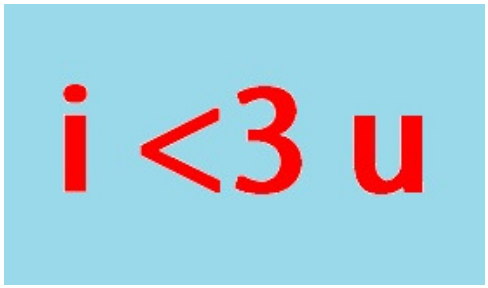
Problem A $3x - 9 \le 18$	Problem B $6w - 1 < -13$	Problem C $2 - \frac{1}{2}b < 5$	Problem D $-0.3p - 2.1 \le -3.6$

8.	How are problem A and problem D similar?
9.	How are problem B and problem C similar?
10.	How are problem A and problem B different from problem C and problem D?

Activity 4II-10: **Solving Two-Step Inequalities** Name:

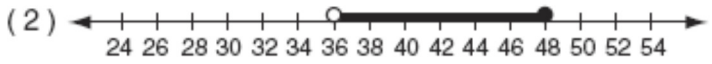
Write an inequality for each problem and solve showing all work on a separate sheet of paper.

1.	Which value of x is in the solution set of the inequality $-2x+5>17$? -8 -6 -4 12	
2.	Which value of x is in the solution set of the inequality $-4x+2>10$? -2 2 3 -4	
3.	Which values will not make the inequality $8w-6<42$ true? I. 9 II. 3 III. 5 IV. 6	
4.	Which value of x is in the solution set of the inequality $-2(x-5)<4$? 0 2 3 5	
5.	What is the solution of $3(2m-1)\le 4m+7$?	
6.	What is the solution of $-6x-17\ge 8x+25$?	
7.	Roger is having a picnic for 78 guests. He plans to serve each guest at least one hot dog. If each package, p, contains eight hot dogs, which inequality could be used to determine how many packages of hot dogs Roger will need to buy? $p\ge 78$ $8p\ge 78$ $8+p\ge 78$ $78-p\ge 8$	
8.	The ninth grade class at a local high school needs to purchase a park permit for $250 for their upcoming class picnic. Each ninth grader attending the picnic pays $0.75. Each guest pays $1.25. If 200 ninth graders attend the picnic, which inequality can be used to determine the number of guests, x, needed to cover the cost of the permit? $0.75x-(1.25)(200)\ge 250.00$ $0.75x+(1.25)(200)\ge 250.00$ $(0.75)(200)-1.25x\ge 250.00$ $(0.75)(200)+1.25x\ge 250.00$	
9.	Solve the following inequalities: $5n>-25$, $\frac{r}{6}\le 3$, $\frac{y}{-6}>10$, $\frac{1}{4}n<8$	
10.	Write an inequality for the graph. 	



Activity 4II-11: **Solving Two-Step Inequalities** Name:

Define a variable. Then write an inequality for each problem and solve showing all work.

1.	The sum of twice a number and 5 is at most 15. What are the possible values for the number?
2.	Two-thirds of a number plus 5 is greater than 12. Find the number.
3.	In order to be admitted for a certain ride at an amusement park, a child must be greater than or equal to 36 inches tall and less than 48 inches tall. Which graph represents these conditions? (1)  (2)  (3)  (4) 
4.	Which statement can be modeled by $x + 3 \leq 12$? A Sam has 3 bottles of water. Together, Sam and Dave have at most 12 bottles of water. B Jennie sold 3 cookbooks. To earn a prize, Jennie must sell at least 12 cookbooks. C Peter has 3 baseball hats. Peter and his brothers have fewer than 12 baseball hats. D Kathy swam 3 laps in the pool this week. She must swim more than 12 laps.
5.	Julia has $80. She wants to purchase a nail set for $16 and earrings. She spends the rest of the money on earrings. Each pair of earrings costs $8. Write an inequality for the number of pairs of earrings she can purchase and solve.
6.	Liam brought $28 to the arcade and played games that cost 75 cents each. Laney brought $12 to the arcade and spent $4.25 on a slice of pizza and a Coke. How many games can Liam play so that he leaves with less money than Laney?
7.	Christina goes to the market with $50. She buys some papayas for $20 and spends the rest of the money on bananas. Each banana cost $0.60. Write an inequality for the number of bananas she can purchase and solve.
8.	Billy goes to the store. He has $90. He wants to purchase a leather jacket for $45, a hat for $10, and the rest on jeans. Each pair of jeans costs $35. Write an inequality for the number of jeans he can purchase and solve.
9.	Rebecca bought one goldfish ($32) and one starfish ($12). She spends the rest of her money on guppy fish. She starts with $80. Each guppy costs $6. Write and solve inequality for the number of guppies she can purchase.

Define a variable. Then write an inequality for each problem and solve showing all work on a separate sheet of paper.

1.	Tamara has a cell phone that charges $0.07 per minute plus a monthly fee of $19.00. She budgets $29.50 per month for total cell phone expenses without taxes. What is the maximum number of minutes Tamara could use her phone each month in order to stay within her budget?
2.	An online music club has a one-time registration fee of $13.95 and charges $0.49 to buy each song. If Emma has $50 to join the club and buy songs, what is the maximum number of songs she can buy?
3.	Peter begins his kindergarten year able to spell 10 words. He is going to learn to spell 2 new words every day. Write an inequality that can be used to determine how many days, d, it takes Peter to be able to spell at least 75 words. Use this inequality to determine the minimum number of whole days it will take for him to be able to spell at least 75 words.
4.	Chelsea has $45 to spend at the fair. She spends $20 on admission and $15 on snacks. She wants to play a game that costs $0.65 per game. Write an inequality to find the maximum number of times, x, Chelsea can play the game. Using this inequality, determine the maximum number of times she can play the game.
5.	Melissa wants to spend no more than $300 on school clothes. She spends $75 on a coat and then wants to buy some sweaters that are on special for $10 each. Solve the inequality $75+10s \le 300$ to find the greatest number of sweaters s she can buy.
6.	A small airplane can carry less than 1,050 pounds of luggage and mail. The mail for the day weighs 490 pounds. If each passenger brings 70 pounds of luggage, what is the greatest possible number of passengers that can be taken?
7.	You rent a car and are offered 2 payment options. You can pay $25 a day plus 15 cents a mile or you can pay $10 a day plus 40 cents a mile. For what amount of daily miles will Option A be the cheaper plan? Write and solve an inequality to determine the answer.
8.	Keith and Michelle went out to dinner. The total amount they had to spend on the cost of the meal, including the tip, was to $53.70. If the combined tip came out to $9.60, and each friend spent an equal amount, how much could each friend pay, not including the tip?
9.	Jason is saving up to buy a digital camera that costs $490. So far, he saved $175. He would like to buy the camera 3 weeks from now. Write an inequality to represent at least how much he must save every week to have enough money to purchase the camera. Solve.
10.	Adrian works in New York City and makes $42 per hour. She works in an office and must get her suit dry cleaned every day for $75. If she wants to end up with more than $260 a day, *at least* how many hours must she work?
11.	Erin has $50. She wants to purchase a cell phone ($20) and spend the rest on music CDs. Each music CD costs $8. Write and solve an inequality for the number of music CDs she can purchase.