

# ACCELERATED MATHEMATICS

## CHAPTER 3

### FRACTIONS

#### TOPICS COVERED:

- Divisibility Rules
- Primes & Prime Factorization
- Greatest Common Factor
- Least Common Multiple
- Fraction sense
- Adding and subtracting fractions and mixed numbers
- Equations with adding and subtracting fractions
- Hands-on multiplying fractions
- Multiplying fractions and mixed numbers
- Applications of multiplying fractions
- Hands-on dividing fractions
- Dividing fractions and mixed numbers
- Applications of dividing fractions



**3 OUT OF 2  
PEOPLE  
HAVE  
TROUBLE  
WITH  
FRACTIONS**

**Divisibility Rules**

<b>2</b> A number is divisible by 2 if the ones digit is even.	<b>3</b> A number is divisible by 3 if the sum of its digits is divisible by 3.
<b>4</b> A number is divisible by 4 if its last two digits are divisible by 4.	<b>5</b> A number is divisible by 5 if it ends in 0 or 5.
<b>6</b> A number is divisible by 6 if it is divisible by both 2 and 3.	<b>9</b> A number is divisible by 9 if the sum of its digits is divisible by 9.
<b>10</b> A number is divisible by 10 if its last digit is 0.	

Circle the numbers that are divisible by 2.

324	243	432	423	234
196	825	4374	9701	65250

Circle the numbers that are divisible by 3.

48	75	76	77	78
761	762	763	764	765
46	51	913	834	7085

Circle the numbers that are divisible by 4.

934	924	944	954	964
732	742	752	762	772

Circle the numbers that are divisible by 5.

354	355	375	380	385
650	605	506	560	1056

Circle the numbers that are divisible by 6.

78	62	3054	5553	24718
69300	762	765	96	104

Circle the numbers that are divisible by 9.

377	378	387	837	827
4876	5876	5976	9567	5796

Circle the numbers that are divisible by 10.

100	75	23	60	108
120	245	250	380	387

1.	The Southlake Carroll Marching Band is getting ready to perform at halftime of the football game. With 216 musicians, can the marching band form equal rows of 3? of 4? of 5? of 6? of 9?	
2.	True or false: All numbers divisible by 5 are also divisible by 10.	
3.	True or false: All numbers divisible by 10 are also divisible by 5.	
4.	True or false: All numbers divisible by 9 are also divisible by 3.	
5.	A giant pizza is divided into 18 pieces. What are the different numbers of people you can divide it among so that there are no pieces left over?	
6.	What is the smallest number you find that is divisible by 2, 3, 5, 6, 9, and 10?	

**Complete the table. Answer Y (yes) or N (no) for each box.**

	Number	Divisible by 2	Divisible by 3	Divisible by 4	Divisible by 5	Divisible by 6	Divisible by 9	Divisible by 10
7.	324							
8.	475							
9.	525							
10.	3951							
11.	4230							
12.	7803							
13.	9360							
14.	11,235							
15.	15,972							
16.	23,409							

17.	Marty said to Doc, “So we are going to travel back in time. What year did you set the Delorean for?” Doc replied, “I can’t remember exactly, but I do remember the following: If you divide the year by 2, you’ll get a remainder of 1. If you divide the year by 3, 4, 5, 6, 7, or 9, you’ll also get a remainder of 1.” “What about 8? Do you also get a remainder of 1?” “No,” said Doc. Marty then knew which year they were off to. Which year?	
18.	The page numbers of a book are numbered 1 to 300. How many page numbers meet these conditions: A. The page numbers have the digit 5 and are also divisible by 5. B. The page numbers contain the digit 5 but are not divisible by 5. C. The page numbers do not have a 5 but are divisible by 5.	

1. Circle all the prime numbers in the first row.
2. Draw a line through the first column (except for 2) and through the third and fifth columns.
3. Draw a line through the second column (except for 3).
4. Draw a diagonal line from the 5 in the top row (not including the 5) to the 5 in the side column. Repeat with diagonal lines between the pairs of 5's in the side columns.
5. Draw a diagonal line down and to the right between the first 7 in the left side column to the first 7 in the right side column. Do this again for the second 7's.
6. Circle any number that does not have a line through it.
7. Explain why you are left with just the primes.

Side Column	First	Second	Third	Fourth	Fifth	Sixth	Side Column
	2	3	4	5	6	7	
7A	8	9	10	11	12	13	
	14	15	16	17	18	19	5A
	20	21	22	23	24	25	
5	26	27	28	29	30	31	
	32	33	34	35	36	37	
	38	39	40	41	42	43	
	44	45	46	47	48	49	5B
7B	50	51	52	53	54	55	7A
5A	56	57	58	59	60	61	
	62	63	64	65	66	67	
	68	69	70	71	72	73	
	74	75	76	77	78	79	5C
	80	81	82	83	84	85	
5B	86	87	88	89	90	91	
	92	93	94	95	96	97	7B
	98	99	100	101	102	103	

5C

# The World Has A New Largest-Known Prime Number

There's a new behemoth in the ongoing search for ever-larger prime numbers — and it's nearly 25 million digits long.

A prime is a number that can be divided only by two whole numbers: itself and 1. The newly discovered number is what's known as a Mersenne prime, named for a French monk named Marin Mersenne who studied primes some 350 years ago.

Mersenne primes have a simple formula:  $2^n - 1$ . In this case, "n" is equal to 82,589,933, which is itself a prime number. If you do the math, the new largest-known prime is a whopping 24,862,048 digits long.

We would write the number out for you, but it would fill up thousands of pages, give or take.

The latest Mersenne prime comes courtesy of a project started in 1996 called the [Great Internet Mersenne Prime Search](#), in which people download special software that runs in the background on their computers. A computer owned by Patrick Laroche of Ocala, Fla., discovered the number on Dec. 7, 2018 and mathematicians have spent the past two weeks verifying the calculations. It's more than a million and a half digits longer than the previous largest known prime, discovered about a year ago by a computer in Germantown, Tenn.

Why should you care about Mersenne primes? They've proven useful in [generating reasonably random numbers](#), but the best answer might be simply because they're there — and they're majestic.

A **prime number** is a number with only 2 factors. **Composite numbers** have more than 2 factors.

<del>1</del>	2	3	<del>4</del>	5	<del>6</del>	7	<del>8</del>	<del>9</del>	<del>10</del>
11	<del>12</del>	13	<del>14</del>	<del>15</del>	<del>16</del>	17	<del>18</del>	19	<del>20</del>
<del>21</del>	<del>22</del>	23	<del>24</del>	<del>25</del>	<del>26</del>	<del>27</del>	<del>28</del>	29	<del>30</del>
31	<del>32</del>	<del>33</del>	<del>34</del>	<del>35</del>	<del>36</del>	37	<del>38</del>	<del>39</del>	<del>40</del>
41	<del>42</del>	43	<del>44</del>	<del>45</del>	<del>46</del>	47	<del>48</del>	<del>49</del>	<del>50</del>
<del>51</del>	<del>52</del>	53	<del>54</del>	<del>55</del>	<del>56</del>	<del>57</del>	<del>58</del>	59	<del>60</del>
61	<del>62</del>	<del>63</del>	<del>64</del>	<del>65</del>	<del>66</del>	67	<del>68</del>	<del>69</del>	<del>70</del>
71	<del>72</del>	73	<del>74</del>	<del>75</del>	<del>76</del>	<del>77</del>	<del>78</del>	79	<del>80</del>
<del>81</del>	<del>82</del>	83	<del>84</del>	<del>85</del>	<del>86</del>	<del>87</del>	<del>88</del>	89	<del>90</del>
<del>91</del>	<del>92</del>	<del>93</del>	<del>94</del>	<del>95</del>	<del>96</del>	97	<del>98</del>	<del>99</del>	<del>100</del>

Using your 100 Board, answer the following questions.

1.	What is the smallest prime number that is greater than 30?	
2.	What is the smallest prime number that is greater than 50?	
3.	5 and 7 are called twin primes because they are both primes and they differ by two. List all twin primes between 1 and 100.	
4.	Find 5 composite numbers in a row.	
5.	On Activity 3-3, why didn't we have to keep going and cross out all multiples of 9?	
6.	Which of the primes 2, 3, 5, and 7 divide into 84?	

Cross out the boxes containing composite numbers to discover the hidden message.

D	P	I	R	V	I	M	P	S	K	S	O	Z	R	S
7	6	2	8	19	11	12	60	3	9	14	59	35	11	37
Q	A	R	M	E	S	D	M	I	V	H	I	N	E	A
4	3	31	25	23	10	29	12	41	97	100	23	83	13	12
B	U	R	T	T	F	O	A	I	C	T	R	O	R	S
71	2	35	3	27	43	42	37	64	7	5	45	13	11	71
N	E	U	M	A	S	F	G	O	R	K	E	Q	T	D
9	14	69	32	17	87	48	75	20	19	9	97	8	27	57
F	R	C	I	M	E	T	K	N	D	L	N	I	E	R
67	2	16	89	18	7	12	9	17	73	67	49	59	29	83

Why isn't zero composite? For a number to be composite it has to be able to be written as a product of two factors, neither of which is itself. Zero has an infinite number of factors; however they are all multiplied by zero to equal zero.

**Create a factor tree for each number. Write your answer using exponents. Show all work/answers on separate paper.**

1.	144	2.	110	3.	280
4.	123	5.	200	6.	900
7.	108	8.	1500	9.	324

10.	What is the maximum number of prime numbered dates in any two consecutive months?
11.	On a prime date both the month and the date are prime numbers. For example, 2/3 is a prime date. How many prime dates occur this year?
12.	I am a two-digit prime number. The number formed by reversing my digits is also prime. My ones digit is 4 less than my tens digit. What number am I?
13.	Use the prime factorizations of these numbers to find the seventh number in the pattern: 8, 18, 32, 50, 72, ...

**Find the GCF of each set of numbers.**

14.	28, 32	15.	48, 56
16.	25, 30	17.	72, 84, 132
18.	20, 28, 36	19.	126, 168, 210

20.	A lady wants to buy plants for her garden and wants to plant them in rows with the same number of plants in each row. Which of these would give her the most choices? 8 flats of 6 plants OR 7 flats of 8 plants OR 5 flats of 10 plants
21.	What numbers between 50 and 100 have the greatest number of factors?
22.	What calendar date(s) has the most factors? have exactly 3 factors?
23.	Rebecca's little sister Tina has 48 yellow blocks and 40 green blocks. Tina builds some number of towers using all 88 blocks. What is the greatest number of identical towers that Tina can build? How many green and yellow blocks are in each tower?
24.	Mr. Mangham has some cookies. They can be divided evenly among 9 students. They can also be divided evenly among 6 students. What are two possibilities for the number of cookies?

You can use a triple Venn diagram to find the GCF and LCM of set of monomials.

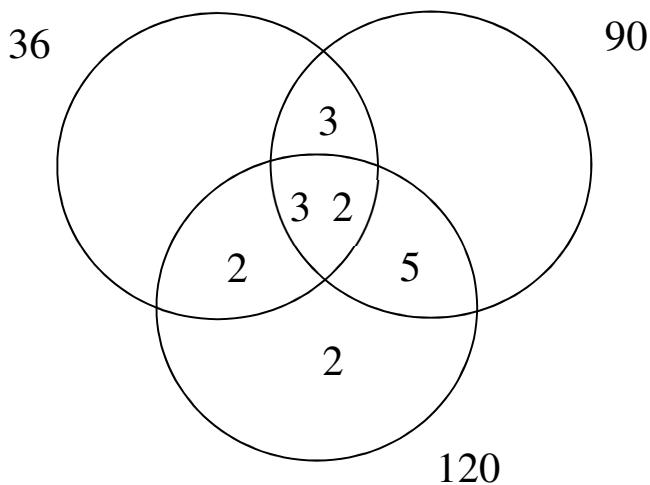
Below is an example of how to complete a triple Venn diagram.

Find the GCF and LCM of 36, 90, and 120.

The prime factorization of 36 =  $3 \cdot 3 \cdot 2 \cdot 2$

The prime factorization of 90 =  $5 \cdot 3 \cdot 3 \cdot 2$

The prime factorization of 120 =  $5 \cdot 3 \cdot 2 \cdot 2 \cdot 2$



GCF = 6 because all three numbers have both a 2 and a 3 in common. 2 times 3 equals 6.

LCM =  $5 \cdot 3 \cdot 3 \cdot 2 \cdot 2 \cdot 2 = 360$  because you multiply all terms in the diagram. Notice an easy way to do this is to take the 120 circle and just multiply it by the only number, 3, sitting outside this circle.



Multiple comes from “multi” meaning many and “pli” meaning fold. Fold a piece of paper in half, in half again, and again. The resulting number of pieces is eight times the number of original pieces. Remember: Every number has a multitude of multiples!

**Find the LCM for each set of numbers or monomials.**

1.	2, 4, 5		2.	5, 6, 18	
3.	12, 16, 30		4.	12, 16	
5.	5, 10, 20		6.	14, 22	
7.	6, 8, 10		8.	18, 30, 50	
9.	6, 8, 36		10.	24, 16, 30	
11.	15, 20, 30		12.	12, 25, 18	

13.	The GCF of two numbers is 850. Neither number is divisible by the other. What is the smallest that these two numbers could be?	
14.	The GCF of two numbers is 479. One number is even and the other number is odd. Neither number is divisible by the other. What is the smallest that these two numbers could be?	
15.	Ms. Wurst and Mr. Pop have donated a total of 90 hot dogs and 126 small cans of fruit juice for a math class picnic. Each student will receive the same amount of refreshments. What is the greatest number of students that can attend the picnic? How many cans of juice will each student receive? How many hot dogs will each student receive?	
16.	At one store hot dogs come in packages of eight. Hot dog buns come in packages of twelve. What is the least number of packages of each type that you can buy and have no hot dogs or buns left over?	
17.	Tongue Tickler Tooth Paste comes in two sizes: 9 oz. for \$0.89 12 oz. for \$1.19 A. What is the LCM of 9 and 12? B. If you bought that much toothpaste in 9-oz. tubes, how much would it cost? C. If you bought that much toothpaste in 12-oz. tubes, how much would it cost? D. Which tube gives you more Tongue Tickler Toothpaste for the money?	

**Advanced Venn Diagram Problem**

18.	Every red car at an auto show was a sports car. Half of all the blue cars were sports cars. Half of all sports cars were red. There were forty blue cars and thirty red cars. How many sports cars were neither red nor blue?	
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**Solve.**

1.	In the school kitchen during lunch, the timer for pizza buzzes every 14 minutes; the timer for hamburger buns buzzes every 6 minutes. The two timers just buzzed together. In how many minutes will they buzz together again?
2.	Two neon signs are turned on at the same time. One blinks every 4 seconds; the other blinks every 6 seconds. How many times per minute do they blink on together?
3.	A company ships in two different sized boxes. One box is 5 inches long and the other is 8 inches long. What is the shortest length crate the company can use to ship its product in either sized box without having extra space?
4.	A construction company uses 15-foot long concrete blocks for the width and 45-foot long blocks for the length of any rectangular building. What is the shortest length square building the company could construct?
5.	On Southlake Highway there are rest stops every 50 miles. Lauren's family stops at the rest stops every 150 miles. Kyle's family stops every 250 miles. The two families began their trips from the same place. What is the shortest distance the two families must drive before they stop at the same rest stop?
6.	Jim is planning a party for his class. He is going to buy hot dogs and hot dog buns. The hot dogs come 10 to a package. The buns come 8 to a package. He doesn't want any hot dogs without buns and doesn't want any buns or hot dogs left over. How many packages of each should he buy?
7.	Stephanie's English teacher gives a test every 3 school days. Her mathematics teacher gives a test every 4 school days and her social studies teacher gives a test every 5 school days. She knows that she could have all 3 tests on the same day. How often will she have three tests on one day?
8.	Lance is making cookies for the bake sale. He wants to use up the ingredients he is going to buy to make cookies. The recipe calls for 1 cup of chocolate chips, 1 cup of pecans, and 1 cup of coconut. The chocolate chip package contains 2 cups of chips. The pecan package contains 4 cups of pecans. The coconut package contains 7 cups of coconut. How many packages of each will he need to buy to use up all of the ingredients? How many batches of cookies will that make?
9.	Danielle thinks of two numbers that are multiples of 9. The product of the two digits (neither a 9) of either of her numbers is also a multiple of 9. What are her numbers?
10.	Danny is planning on having children make ladybugs for their craft this week. Each ladybug needs 1 Styrofoam hemisphere, two fuzzy stems for antenna, and six regular stems for legs. The Styrofoam hemispheres are 5 in a package, the fuzzy stems are 6 in a package, and the regular stems are 12 in a package. How many packages of each are needed to make ladybugs with no left over parts? How many ladybugs can be made with these packages?
11.	A band of pirates divided 185 pieces of silver and 148 gold coins. Since pirates are known to be fair about sharing equally, how many pirates were there?
12.	The number 6 has exactly four whole number divisors: 1, 2, 3, and 6. What is the smallest number with exactly five whole number divisors?

**Juniper Green – Round 1**

- Rules of the game:
1. Two players play at a time. The first player selects an even number.
  2. On each turn, a player selects any remaining number that is a factor or a multiple of the number just selected by his or her opponent.
  3. The first player who cannot select a number loses.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

**Juniper Green – Round 2**

Rules of the game: The rules this time are very similar to the first game, except you and your partner are now working together. Try to stay alive as long as possible by crossing out as many numbers as possible. The game is over when a player cannot select another number.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Find three fractions equivalent to each of the following.

1.	$\frac{2}{3}$		2.	$\frac{5}{4}$	
3.	$\frac{1}{7}$		4.	$\frac{4}{12}$	

Write each fraction in simplest form.

5.	$\frac{28}{36}$		6.	$\frac{81}{90}$		7.	$\frac{8}{21}$		8.	$\frac{14}{35}$	
9.	$\frac{63}{108}$		10.	$\frac{45}{48}$		11.	$\frac{12}{27}$		12.	$\frac{17}{51}$	

Write each improper fraction as a mixed number.

13.	$\frac{17}{5}$		14.	$\frac{17}{6}$		15.	$\frac{21}{10}$		16.	$\frac{25}{4}$	
17.	$\frac{20}{3}$		18.	$\frac{15}{6}$		19.	$\frac{30}{8}$		20.	$\frac{100}{75}$	

Write each whole or mixed number as an improper fraction.

21.	$3\frac{3}{4}$		22.	4		23.	$5\frac{5}{6}$		24.	$1\frac{9}{10}$	
25.	$3\frac{11}{12}$		26.	$4\frac{7}{8}$		27.	$5\frac{4}{15}$		28.	6	

Order each set of fractions from greatest to least.

29.	$\frac{3}{4}, \frac{5}{8}, \frac{1}{2}$		30.	$\frac{5}{8}, \frac{5}{6}, \frac{2}{3}$	
31.	$\frac{1}{2}, \frac{5}{12}, \frac{2}{3}$		32.	$\frac{3}{5}, \frac{2}{3}, \frac{7}{12}$	
33.	$\frac{1}{2}, \frac{3}{5}, \frac{3}{8}$		34.	$\frac{7}{8}, \frac{3}{4}, \frac{13}{16}$	
35.	$\frac{8}{9}, \frac{3}{4}, \frac{5}{6}, \frac{2}{3}$		36.	$\frac{4}{5}, \frac{13}{16}, \frac{7}{8}, \frac{31}{40}$	

**Use your fraction sense to solve each problem.**

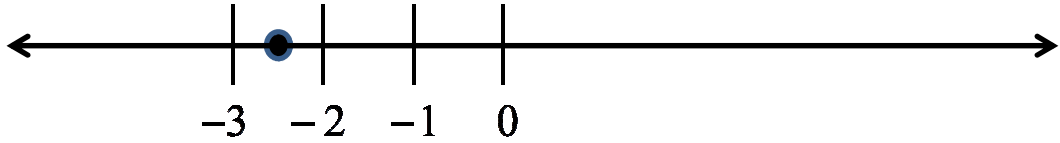
1.	Name a fraction between $\frac{1}{3}$ and $\frac{2}{3}$ .	
2.	Name a fraction between $\frac{3}{5}$ and $\frac{4}{5}$ .	
3.	Name a fraction between $\frac{1}{4}$ and $\frac{1}{2}$ whose denominator is 16.	
4.	Name a fraction between $\frac{2}{3}$ and $\frac{3}{4}$ whose denominator is 10.	
5.	Name a fraction that is halfway between $\frac{2}{9}$ and $\frac{5}{9}$ .	
6.	Name a fraction that is halfway between $\frac{2}{3}$ and $\frac{4}{5}$ .	
7.	How many fractions are there between one-fourth and one-half?	

**Use the clues to discover the identity of the mystery fraction.**

8.	My numerator is 6 less than my denominator. I am equivalent to $\frac{3}{4}$ .	
9.	The GCF of my numerator and denominator is 5. I am equivalent to $\frac{4}{6}$ .	
10.	My numerator and denominator are prime numbers. My numerator is one less than my denominator.	
11.	My numerator and denominator are prime numbers. The sum of my numerator and denominator is 24.	
12.	My numerator is divisible by 3. My denominator is divisible by 5. My denominator is 4 less than twice my numerator.	
13.	My numerator is divisible by 3. My denominator is divisible by 5. My denominator is 3 more than twice my numerator.	
14.	My numerator is a prime number. The GCF of my numerator and denominator is 2. I am equivalent to one-fifth.	

**Add or Subtract. Write each answer in simplest form.**

15.	$4\frac{5}{8} + 2\frac{1}{4} =$		16.	$8\frac{3}{5} + 2\frac{7}{10} =$	
17.	$5\frac{2}{7} + 3\frac{1}{2} =$		18.	$7\frac{1}{4} + 8\frac{3}{8} =$	
19.	$5\frac{1}{8} - 2\frac{1}{3} =$		20.	$7\frac{6}{7} - 2\frac{8}{14} =$	
21.	$10\frac{4}{9} - 3\frac{9}{10} =$		22.	$12\frac{1}{4} - 8\frac{3}{5} =$	

<b>The Basics of Negative Fractions</b>	
$-2\frac{1}{2}$	<p>The number above represents negative two and one-half.</p> <p>On a number line, the mixed number <math>-2\frac{1}{2}</math> would be placed as below.</p>
	
<p>It is located between the negative 2 and the negative 3. Other ways to think of this number is as</p> $-\left(2\frac{1}{2}\right) \text{ or } -2 + -\frac{1}{2}$	
<p>When converting a negative mixed number to a negative improper fraction, ignore the negative sign, convert like normal, and then place the negative sign back in at the end.</p>	
Ex. 1	$-5\frac{1}{3}$ Three times five equal fifteen plus one equals 16, so $-\frac{16}{3}$ .

**Write each improper fraction as a mixed number.**

1.	$\frac{11}{10}$		2.	$-\frac{7}{6}$		3.	$\frac{9}{8}$		4.	$-\frac{11}{8}$	
5.	$-\frac{15}{8}$		6.	$\frac{21}{4}$		7.	$-\frac{17}{3}$		8.	$\frac{17}{4}$	
9.	$\frac{17}{5}$		10.	$-\frac{17}{6}$		11.	$\frac{21}{10}$		12.	$-\frac{25}{4}$	
13.	$-\frac{20}{3}$		14.	$\frac{15}{6}$		15.	$-\frac{30}{8}$		16.	$\frac{100}{75}$	

**Write each whole or mixed number as an improper fraction.**

17.	$3\frac{3}{4}$		18.	4		19.	$-5\frac{5}{6}$		20.	$1\frac{9}{10}$	
21.	$-2\frac{3}{8}$		22.	$4\frac{7}{8}$		23.	$-2\frac{3}{5}$		24.	6	
25.	$3\frac{11}{12}$		26.	$-2\frac{7}{12}$		27.	$5\frac{4}{15}$		28.	$-2\frac{7}{15}$	
29.	$-1\frac{5}{6}$		30.	$9\frac{3}{5}$		31.	$-6\frac{2}{3}$		32.	$5\frac{5}{9}$	

**Converting fractions to decimals**

<b>Fraction to a decimal</b>	<b>Divide!</b>
	<p><i>Example 1:</i> <math>\frac{3}{5} = 3 \div 5 = 0.6</math></p> <p>If the decimal keeps repeating use bar notation.</p> <p><i>Example 2:</i> <math>\frac{2}{3} = 2 \div 3 = 0.\overline{6}</math></p>

**Write each repeating decimal using bar notation.**

1.	0.22222...		2.	0.41666...		3.	0.54545...	
4.	0.6363...		5.	0.2727...		6.	0.428572428..	

**Express each fraction or mixed number as a decimal. Use bar notation, if necessary.**

7.	$\frac{4}{9}$		8.	$1\frac{7}{18}$		9.	$\frac{5}{7}$	
10.	$2\frac{3}{16}$		11.	$7\frac{1}{18}$		12.	$\frac{8}{11}$	
13.	$4\frac{1}{6}$		14.	$-6\frac{1}{12}$		15.	$14\frac{5}{8}$	
16.	$-\frac{3}{13}$		17.	$-\frac{6}{7}$		18.	$5\frac{8}{9}$	
19.	$8\frac{2}{3}$		20.	$\frac{5}{16}$		21.	$-\frac{9}{11}$	

**Order each set of rational numbers from least to greatest.**

22.	1, 1.1, 1.01, $1.\overline{11}$		23.	2, -2, -2.1, $2.\overline{1}$	
24.	$\frac{2}{3}, \frac{1}{2}, \frac{3}{5}$		25.	$0, -2.1, \frac{3}{2}, \frac{2}{3}$	
26.	$\frac{4}{11}, 0.35, 0.3\overline{6}$		27.	$\frac{3}{4}, \frac{9}{10}, \frac{7}{8}, \frac{2}{3}$	
28.	$-0.3, -\frac{1}{3}, -0.33, -0.35$		29.	$-\frac{5}{6}, -\frac{4}{5}, -0.83, -0.8\overline{01}$	
30.	$0.4\overline{7}, 0.4\overline{74}, 0.4\overline{7}, 47\%, 47.4\overline{4}$				

Now that we have learned about bar notation with decimals here is a serious problem for you:

How do you write  $0.99999999\dots$  That would be  $0.\overline{9}$ , right?

Well,  $0.999999\dots$  repeated forever equals what?

**Would you say that number is equal to 1 or that it is less than 1??? Think about it.**

In ordinary math, this number equals one. Does your head hurt yet?

So how can  $.9999\dots=1$ ?

There are many different proofs of the fact that  $0.9999\dots$  does indeed equal 1. So why does this question keep coming up?

Do you agree that  $0.3333\dots$  is equal to  $\frac{1}{3}$ ?

Remember  $0.9999\dots$  doesn't mean "0.9" or "0.99" or "0.9999" or "0.999 followed by some large but finite (limited) number of 9's".  $0.9999\dots$  never ends. There will *always* be another "9" to tack onto the end of  $0.9999\dots$ . So don't object to  $0.9999\dots = 1$  on the basis of "however far you go out, you still won't be equal to 1", because there is no "however far" to "go out" to; you can *always* go further. "But", some say, "there will always be a difference between  $0.9999\dots$  and 1."

Well, sort of. Yes, at any given stop, at any given stage of the expansion, for any given *finite* number of 9s, there will be a difference between  $0.999\dots 9$  and 1. That is, if you do the subtraction,  $1 - 0.999\dots 9$  will not equal zero. But the point of the "dot, dot, dot" is that there is no end;  $0.9999\dots$  is infinite. There is no "last" digit. So the "there's always a difference" argument betrays a lack of understanding of the infinite.

We have learned that  $\frac{1}{3} = 0.333\dots$  in decimal form.

So  $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 3(\frac{1}{3}) = 1$ . Reasonably then,  $0.333\dots + 0.333\dots + 0.333\dots = 3(0.333\dots)$  should also equal 1.

But  $3(0.333\dots) = 0.999\dots$ . Then  $0.999\dots$  must equal 1.

If two numbers are different, then you can fit another number between them, such as their average. But what number could you possibly fit between  $0.999\dots$  and  $1.000\dots$ ?

To compare and order fractions use the least common denominator (LCD). The LCD is the least common multiple (LCM) of the original denominators.

**Example**      **Order from least to greatest:**  $\frac{2}{3}, \frac{5}{8}, \frac{3}{4}$

- Find the LCD of the denominators: 3, 8, and 4. The LCD is 24.
- Write equivalent fractions.

$$\frac{2}{3} = \frac{16}{24} \quad \frac{5}{8} = \frac{15}{24} \quad \frac{3}{4} = \frac{18}{24}$$

- Order the fractions.

$$\frac{15}{24} < \frac{16}{24} < \frac{18}{24}$$

**Compare the following fractions using <, >, or =.**

1.	$\frac{2}{9}$	$\frac{1}{3}$	2.	$-\frac{5}{6}$	$-\frac{7}{8}$	3.	$\frac{7}{20}$	$\frac{3}{10}$
4.	$-\frac{3}{6}$	$-\frac{4}{11}$	5.	$\frac{2}{3}$	$\frac{4}{6}$	6.	$-\frac{4}{8}$	$-\frac{2}{8}$
7.	$-\frac{3}{7}$	$-\frac{5}{8}$	8.	$\frac{1}{3}$	$\frac{3}{9}$	9.	$\frac{1}{2}$	$\frac{3}{7}$
10.	$\frac{5}{4}$	$\frac{9}{7}$	11.	$-\frac{2}{3}$	$-\frac{7}{10}$	12.	$2\frac{5}{9}$	$2\frac{3}{5}$

**Order each set of fractions from greatest to least.**

13.	$\frac{3}{4}, \frac{5}{8}, \frac{1}{2}$		14.	$-\frac{5}{8}, -\frac{5}{6}, -\frac{2}{3}$	
15.	$-\frac{1}{2}, -\frac{5}{12}, -\frac{2}{3}$		16.	$\frac{4}{5}, \frac{13}{16}, \frac{7}{8}, \frac{31}{40}$	
17.	$-\frac{8}{9}, -\frac{3}{4}, -\frac{5}{6}, -\frac{2}{3}$		18.	$-\frac{7}{8}, -\frac{3}{4}, -\frac{13}{16}$	

**Order each set of fractions from least to greatest.**

19.	$-\frac{9}{14}, -\frac{17}{35}, -\frac{1}{2}, -\frac{7}{10}$		20.	$\frac{2}{3}, \frac{2}{7}, \frac{13}{14}, \frac{5}{6}$	
21.	$\frac{2}{3}, \frac{4}{5}, \frac{2}{10}, \frac{5}{7}$		22.	$-\frac{23}{52}, -\frac{12}{13}, -\frac{7}{26}, -\frac{7}{8}$	

Example:  $-6\frac{2}{3} - (-8\frac{1}{2})$  First, change the problem to add the opposite. Then ask, “Are there more positives or negatives?” There more positives, so the answer will be positive. How many more positives?

$$\begin{array}{r} 8\frac{1}{2} \\ -6\frac{2}{3} \\ \hline = 1\frac{5}{6} \end{array} \quad \text{So, the answer is } +1\frac{5}{6}$$

Solve each equation. Write the solution in simplest form.

1.	$k = 7\frac{5}{6} - 2\frac{3}{4}$		2.	$\frac{11}{12} - \frac{1}{16} = m$	
3.	$9\frac{7}{8} + 2\frac{1}{6} = n$		4.	$a = 3\frac{1}{8} - \frac{7}{8}$	
5.	$w = \frac{5}{8} - \frac{9}{16}$		6.	$b = 3 - \frac{3}{8}$	
7.	$r = -\frac{5}{12} + \frac{3}{8}$		8.	$d = 2\frac{7}{8} - 6$	
9.	$x = -\frac{3}{10} + \left(-\frac{2}{5}\right)$		10.	$q = 2\frac{2}{3} + \left(-4\frac{1}{4}\right)$	
11.	$d = -3\frac{3}{5} - \frac{9}{10}$		12.	$v = -2\frac{1}{5} - 3\frac{3}{4}$	
13.	$\left(-4\frac{4}{6}\right) - \left(-2\frac{1}{3}\right) = w$		14.	$b = 3\frac{2}{3} - 8$	
15.	$d = 3\frac{3}{4} - 7\frac{2}{3}$		16.	$p = -10\frac{2}{9} - \left(-3\frac{1}{3}\right)$	

Evaluate each expression if  $a = \frac{4}{9}$ ,  $b = -\frac{2}{3}$ , and  $c = -3\frac{7}{18}$ . Write the solution in simplest form.

17.	$a + c$		18.	$b - a$		19.	$c + b$	
20.	$c - a + b$		21.	$a + b + c$		22.	$c - a - b$	

Evaluate each expression if  $x = \frac{3}{8}$ ,  $y = 2\frac{7}{12}$ , and  $z = -\frac{5}{6}$ . Write the solution in simplest form.

23.	$x + y + z$		24.	$x - z$		25.	$y - (-z)$	
26.	$x + z$		27.	$y - x$		28.	$z + (-y)$	

<p>For each level, draw pictures to represent the problem. Look for the “rule” that works based on the answers in the pictures.</p>		
<p><b>Multiplication</b> <i>Level 0</i></p>	$4 \bullet 3$ $2 \bullet 6$	<p>4 groups of 3 2 groups of 6</p>
<p><b>Multiplication</b> <i>Level 1</i></p>	$2 \bullet \frac{1}{3} = \frac{2}{3}$	<p>Sample of how to read: Five groups of one-sixth</p>
	$3 \bullet \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$	
<p><b>Multiplication</b> <i>Level 2</i></p>	$2 \bullet \frac{2}{6} = \frac{4}{6} = \frac{2}{3}$	<p>Sample of how to read: Three groups of two-fifths</p>
	$3 \bullet \frac{2}{5} = \frac{6}{5} = 1\frac{1}{5}$	
<p><b>Multiplication</b> <i>Level 3</i></p>	$\frac{1}{4} \bullet 8 = 2 \text{ (or } \frac{8}{4} \text{)}$	<p>Sample of how to read: One-fifth of a group of ten</p>
	$\frac{2}{5} \bullet 10 = 4 \text{ (or } \frac{20}{5} \text{)}$	
<p><b>Multiplication</b> <i>Level 4</i> <b>Multiplication</b></p>	$\frac{1}{2} \bullet \frac{4}{5} = \frac{2}{5} = \frac{4}{10}$	<p>Sample of how to read: One-half of a group of four-fifths</p>
	$\frac{2}{3} \bullet \frac{1}{4} = \frac{1}{6} = \frac{2}{12}$	
	$\frac{2}{5} \bullet \frac{3}{4} = \frac{3}{10} = \frac{6}{20}$	
<p><b>Multiplication</b> <i>Level 5</i> <b>Mixed Numbers</b></p>	$1\frac{1}{2} \bullet 2\frac{1}{2}$ $1\frac{1}{3} \bullet 4\frac{1}{2}$	<p>You can not simply multiply the whole numbers and then the fractions! <b>Show distributive property = drawing with pictures!</b></p>
<p><b>Multiplication</b> <i>Level 6</i> <b>Negatives</b></p>		<p>Include negative values</p>
<p><b>Multiplication</b> <i>Level 7</i> <b>Simplify before you multiply</b></p>	$\frac{4}{5} \bullet \frac{5}{8}$	<p>The commutative property allows simplifying before multiplying.</p>

1. Multiplication is the same as repeated addition when you add the same number again and again.
2. Times means “groups of.”
3. A multiplication problem can be shown as a rectangle.
4. You can reverse the order of the factors and the product stays the same.
5. When you multiply two whole numbers, the product is larger than the factors unless one of the factors is zero or one.
6. When you multiply two positive numbers, the product is larger than the factors unless one of the factors is zero or a fraction smaller than one.

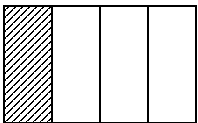
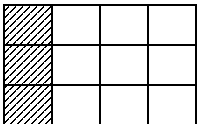
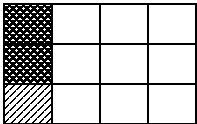


1. Four times three means four groups of three. Three times four means three groups of four.
2. The problem three times four can be shown by a three by four rectangle.
3. The problem one-half times one-half can be shown by a two-by-two rectangle representing one whole.

Draw a picture for  $\frac{3}{4} \bullet \frac{2}{5}$

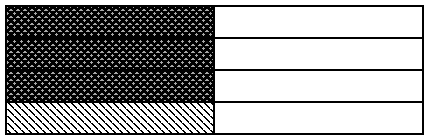
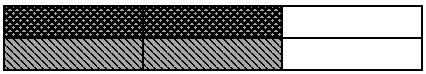

$4\frac{1}{2} \bullet 2\frac{3}{4}$ . Estimate the answer. Use the distributive property.

$3\frac{1}{2} \bullet \frac{2}{3}$ . Estimate the answer.

$2\frac{1}{2} \bullet 5\frac{1}{4} = 10\frac{1}{8}$ . Why not?

<p>Example #1: You can model <math>\frac{2}{3}</math> of <math>\frac{1}{4}</math>.</p> <p>Show <math>\frac{1}{4}</math>. </p> <p>Divide into thirds. </p> <p>Shade <math>\frac{2}{3}</math> of the <math>\frac{1}{4}</math>. </p> <p><math>\frac{2}{3}</math> of <math>\frac{1}{4} = \frac{2}{12} = \frac{1}{6}</math></p>	<p>Example #2: Model <math>\frac{1}{2}</math> of 5.</p> <p>Show 5. </p> <p>Find <math>\frac{1}{2}</math> of each. </p> <p>Combine the 5 halves.</p> <p><math>\frac{1}{2}</math> of 5 = <math>\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{5}{2} = 2\frac{1}{2}</math></p>
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What multiplication expression is represented by each model?

1. 	2. 	3. 
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Draw a model to represent each product.

4. $\frac{1}{6}$ of $\frac{3}{4}$	5. $\frac{2}{5}$ of $\frac{1}{2}$
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Find each product. You do not have to simplify improper fractions into mixed numbers.

6. $\frac{1}{9}$ of $\frac{2}{3}$	7. $\frac{2}{7} \cdot \frac{1}{2}$	8. $\frac{5}{8} \cdot 6$
9. $\frac{3}{8}$ of $\frac{7}{10}$	10. $\frac{3}{4} \cdot \frac{1}{9}$	11. $\frac{2}{9}$ of 8
12. $\frac{1}{3}$ of 2	13. $\frac{5}{9}$ of 4	14. $\frac{3}{4}$ of $\frac{2}{5}$
15. $\frac{1}{3}$ of $\frac{1}{5}$	16. $\frac{5}{8} \cdot \frac{1}{2}$	17. $\frac{4}{9} \cdot \frac{3}{4}$
18. $6 \cdot \frac{2}{3}$	19. $\frac{3}{5} \cdot 10$	20. $12 \cdot \frac{5}{6}$
21. $e = \frac{6}{7}$ of $\frac{8}{15}$	22. $\frac{5}{12} \cdot 10 = t$	23. $\frac{3}{14} \cdot \frac{2}{9} = c$
24. $d = \frac{4}{9} \cdot \frac{15}{16}$	25. $\frac{3}{10} \cdot \frac{5}{8} = k$	26. $h = \frac{3}{7} \cdot 35$
27. $n = \frac{3}{20} \cdot \frac{5}{6}$	28. $\frac{3}{4} \cdot \frac{8}{9} = t$	29. $f = \frac{5}{12} \cdot \frac{4}{15}$

Write true or false.

30. $\frac{3}{4} \cdot \frac{2}{3} < \frac{1}{2}$	31. $\frac{13}{14} \cdot \frac{1}{2} > \frac{13}{14}$
32. $\frac{3}{8} \cdot \frac{4}{5} > \frac{3}{8}$	33. $\sqrt{\frac{5}{7}} < \frac{5}{7}$

To multiply mixed numbers, first convert each to an improper fraction. Then multiply the fractions.

**For all problems on this page, show all work on a separate sheet of paper. Leave your answer as an improper fraction for all problems below.**

Find each product.

1.	$3\frac{1}{5} \cdot 3\frac{3}{4}$	2.	$9 \cdot 4\frac{2}{3}$	3.	$2\frac{5}{6} \cdot 4\frac{1}{2}$
4.	$\frac{4}{7} \cdot 3\frac{1}{9}$	5.	$1\frac{3}{8} \cdot 2\frac{2}{7}$	6.	$4\frac{1}{6} \cdot \frac{9}{10}$
7.	$3\frac{3}{4} \cdot 2\frac{4}{5}$	8.	$\frac{5}{7} \cdot 4\frac{3}{8}$	9.	$20 \cdot 1\frac{2}{5}$

Solve each equation.

10.	$2\frac{4}{9} \cdot \frac{6}{11} = s$	11.	$p = -1\frac{1}{8} \cdot 3\frac{3}{7}$	12.	$-\frac{6}{7} \cdot 2\frac{5}{12} = x$
13.	$d = 14 \cdot -1\frac{3}{4}$	14.	$-5\frac{2}{5} \cdot -\frac{8}{9} = t$	15.	$3\frac{3}{5} \cdot 2\frac{2}{9} = a$
16.	$a = -5\frac{1}{3} \left( -\frac{2}{3} \right)$	17.	$g = -2\frac{1}{4} \left( -\frac{1}{3} \right)$	18.	$s = 7\frac{1}{3} \cdot \frac{6}{11}$
19.	$j = \frac{1}{4} + \frac{1}{3} \cdot \frac{1}{2}$	20.	$b = 3 \left( \frac{4}{5} \right)^2$	21.	$c = -1 \left( -\frac{3}{5} \right)^2$
22.	$-4\frac{1}{3} \cdot \left( \frac{2}{3} \right) + 3\frac{1}{6}$	23.	$-4 - 3\frac{1}{2} \left( \frac{5}{8} \right)$	24.	$-5\frac{1}{2} - 7\frac{1}{3} \cdot \frac{6}{11}$

25.	The weight of an object on Venus is about $\frac{9}{10}$ of its weight on Earth. The weight of an object on Mars is about $\frac{2}{5}$ of its weight on Earth. What is the difference in the weight of a 170 pound person on Venus compared to Mars?
26.	A bulletin board measures 45 feet wide and 18 feet high. Eight posters measuring $2\frac{1}{2}$ feet wide and $3\frac{1}{4}$ feet high are placed on the bulletin board as well as a border covering a total area of 25 square feet. What fraction of the bulletin board is still available for other items?
27.	What fraction is located $\frac{3}{4}$ of the distance between $1\frac{2}{3}$ and $4\frac{1}{2}$ .

**CHOCOLATE BANANA RAISIN COOKIES**

(Makes 24 cookies)

<b>Semi-Sweet Chocolate</b>	$\frac{3}{8}$ of a pound	<b>Butter</b>	$\frac{3}{4}$ of a cup
<b>Flour</b>	$2\frac{1}{8}$ cups	<b>Sugar</b>	$\frac{1}{2}$ of a cup
<b>Cocoa Powder</b>	$\frac{1}{4}$ cup	<b>Light-Brown Sugar</b>	$\frac{1}{2}$ of a cup
<b>Baking Powder</b>	2 teaspoons	<b>Chocolate Extract</b>	1 teaspoon
<b>Salt</b>	$\frac{1}{4}$ of a teaspoon	<b>Eggs</b>	2 large
<b>Mashed Bananas</b>	$3\frac{1}{3}$ medium size	<b>Pecans</b>	$1\frac{1}{2}$ cups
<b>White Chocolate</b>	$\frac{5}{8}$ of a pound	<b>Raisins</b>	1 cup

You gather the ingredients above. Assume each problem uses the ingredients as they start above (do not use the remaining amount for the next problem.) Determine whether the following problems require **subtraction, multiplication, or both**. Then solve. Show all work on a separate sheet of paper.

		Correct Operation(s)	Answer
1.	Nicole ate $\frac{1}{5}$ of a pound of semi-sweet chocolate for breakfast. How much semi-sweet chocolate is left?		
2.	Carter ate $\frac{1}{5}$ of the semi-sweet chocolate with his broccoli soup. How much semi-sweet chocolate did he eat?		
3.	Brandon ate $\frac{1}{5}$ of the semi-sweet chocolate while standing on his head. How much semi-sweet chocolate was left?		
4.	Grant accidentally threw $\frac{3}{4}$ of the flour in the trash. How much flour did he throw in the trash?		
5.	Brooke gave $\frac{3}{4}$ of a cup of flour to her friend Eric. How much flour was left?		
6.	Jared put $\frac{3}{4}$ of the flour on Connor's face. How much flour is not on Connor's face?		

7.	Bailey made 60 cookies, $2\frac{1}{2}$ times the normal recipe, so that her sisters could have some also. How many mashed bananas did she use?		
8.	Emily does not like pecans so she uses only $\frac{2}{7}$ of the normal amount. How many cups of pecans does she use?		
9.	One-third of a pound of white chocolate magically disappears, but some people say Crissy took it. How much white chocolate was left?		
10.	Three-fourths of the butter somehow turns up sitting on Andrew's head. How much butter is left for the recipe?		
11.	Chris has determined that eating small amount of cocoa powder can help you calculate pi more accurately. If Chris ate $\frac{1}{16}$ of a cup of the powder, how much was left to use in the recipe?		
12.	Chad loves pi also. He accidentally takes $\frac{2}{9}$ of the light-brown sugar instead. How much light-brown sugar is left?		
13.	Justin took $\frac{1}{32}$ of a teaspoon of salt and threw it over his shoulder, accidentally hitting Valerie in the face. How much of the salt did not hit Valerie?		
14.	Haley loves raisins. She took $\frac{8}{9}$ of the raisins to create a picture of Raisin Guy. How much of a cup of raisins did she use?		
15.	Michael loves to mix things up so he put the pecans, butter, and both sugars together. He then threw $\frac{4}{5}$ of this mixture directly at Danielle's face. How much of the mixture was still left for Michael to throw at Christen?	Addition PLUS	
16.	Brittany, like, loves to put extra butter on everything. So, like, she uses $5\frac{1}{2}$ times the normal amount of butter in the recipe. How much, like, butter did she use?		
17.	Heather lost $\frac{2}{3}$ of the mashed bananas while she was helping Crissy and Haley learn their dance. How many mashed bananas did she lose?		
18.	Leslie found ants in $\frac{8}{9}$ of a cup of flour. How many cups of flour did not contain ants?		

In order to demonstrate your understanding of the concept of multiplication of fractions, show how to find each of the following products visually. Make a diagram or sketch on graph paper.

1.	$4 \cdot \frac{2}{3}$	2.	$\frac{3}{4}$ of $\frac{4}{7}$	3.	$\frac{1}{4} \cdot 12$
4.	$\frac{1}{2} \cdot \frac{6}{7}$	5.	$\frac{3}{4}$ of 8	6.	$\frac{1}{4} \cdot \frac{3}{4}$

7.-8. Write two **interesting** word problems that require finding the product of fractions. Solve each of your word problems using visual models.

9.	I am a fraction in simplest form. One-sixth of me is the same as one-half of one-fourth. What fraction am I?
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Solve the following problems mentally.

10.	$\frac{2}{3} \cdot 12$	11.	$3 \cdot \frac{1}{4} \cdot \frac{1}{3}$
12.	$20 \cdot \frac{3}{4}$	13.	$\frac{1}{4} \cdot 11 \cdot 12$
14.	$8\frac{2}{7} \cdot 7$	15.	$12 \cdot 5\frac{1}{2}$
16.	$3\frac{3}{4} \cdot 8$	17.	$12 \cdot 3\frac{1}{2}$

Show how to use a sketch to solve the following problems.

18.	After Kevin spent half of his week's pay on food, then spent one-third of what was left on rent, and then spent one-half of what was left on fun, he had \$20 left of his paycheck. How much money was his week's check?
19.	Mr. Mangham has a class of 32 students. Three-fourths of the students are weird and, of those, seven-eighths are clueless. What fraction of all the students are weird and clueless and how many students are neither weird nor clueless? (This assumes that if you are not weird, you are not clueless.)
20.	In an Algebra class, half of the students are boys. One-third of the students are wearing glasses. Half of the boys are wearing glasses. What fraction of the girls is wearing glasses?

Evaluate each expression if  $a = -\frac{1}{4}$ ,  $b = \frac{5}{6}$ ,  $c = -1\frac{1}{2}$ , and  $d = 2\frac{1}{3}$ .

21.	$18b - 6c$	22.	$a + cd$	23.	$9d + \frac{7}{8}$
24.	$a(c + 4)$	25.	$b(a + 8)$	26.	$d(b + 6)$

I baked a batch of happy-face cookies. You see  $\frac{2}{5}$  of the batch below. How many cookies were in the batch?

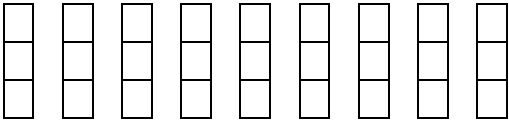


<b>Division</b> <b>Level 0</b> <i>(draw pictures)</i>	$12 \div 3 = 4$ $15 \div 5 = 3$	How many groups of 3 are there in 12? How many groups of 5 are there in 15?
<b>Division</b> <b>Level 1</b> <i>(draw pictures)</i>	$2 \div \frac{1}{2} = 4$	<b>NOTE:</b> When divide by positive fraction, the answer gets bigger. Sample of how to read: How many groups of one-half are in two?
	$4 \div \frac{1}{2} = 8$	
<b>Division</b> <b>Level 2</b> <i>(draw pictures)</i>	$\frac{2}{3} \div \frac{1}{3} = 2$	Sample of how to read: How many groups of one-eighths are in four-eighths?
	$\frac{3}{4} \div \frac{1}{4} = 3$	
	$\frac{5}{6} \div \frac{1}{6} = 5$	
<b>Division</b> <b>Level 3</b> <i>(draw pictures)</i>	$\frac{4}{6} \div \frac{2}{6} = 2$	Sample of how to read: How many groups of three-tenths are in nine-tenths?
	$\frac{6}{8} \div \frac{2}{8} = 3$	
<b>Division</b> <b>Level 4</b> <i>(draw pictures)</i>	$\frac{4}{6} \div \frac{1}{3} = 2 = \frac{4}{2}$	Sample of how to read: How many groups of one-fourth are in nine-twelfths?
	$\frac{8}{12} \div \frac{1}{6} = 4 = \frac{8}{2}$	
<b>Division</b> <b>Level 5</b> <i>(draw pictures)</i>	$3 \div \frac{2}{3} = 4\frac{1}{2}$	Sample of how to read: How many groups of two-thirds are in three?
	$2\frac{3}{4} \div \frac{3}{4} = 3\frac{2}{3}$	
Level 6: Introduce reciprocal rule with mixed numbers.		$3\frac{2}{3} \div 4\frac{1}{4}$
Level 7: Introduce reciprocal rule with negative numbers.		$-2\frac{1}{4} \div -6\frac{1}{5}$
3 rules for dividing fractions: 1. Divide straight across. 2. Find a common denominator 3. Use the reciprocal to make the denominator one		

(Taken from *TEXTEAMS Rethinking Middle School Mathematics: Numerical Reasoning*)

1. You can solve a division problem by subtracting.
2. To divide two numbers,  $a \div b$ , you can think, “How many b’s are in a?”
3. You can check a division problem by multiplying.
4. The division sign means “into groups of.”
5. The quotient tells “how many groups of” there are.
6. You can break the dividend apart to make dividing easier.
7. Remainders can be represented as whole numbers or fractions.
8. If you divide a number by itself, the answer is one.
9. If you divide a number by one, the answer is the number itself.
10. If you reverse the order of the dividend and the divisor, the quotient will be different unless the dividend and divisor are the same number.

Mr. Mangham plans to make small cheese pizzas to sell during the weekends to make some extra money. He has 9 bars of cheddar cheese. How many pizzas can he make if each takes the amount listed in the table? On a piece of notebook paper or computer paper create the five columns shown below. Answer the following questions by completing the row for each question.

	Situation	Think about it (what is the question asking)	Picture It	Write it in symbols	Process it (What do you actually do?)	Solve it
1.	$\frac{1}{3}$ bar of cheese	How many $\frac{1}{3}$ 's are in 9?		$9 \div \frac{1}{3}$	$9 \cdot 3$	
2.	$\frac{1}{4}$ bar of cheese					
3.	$\frac{2}{3}$ bar of cheese					

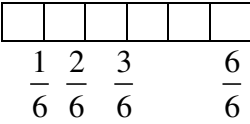
Mr. Mangham still needs more money so he is going to sell small bags of coffee. He buys a large twelve-pound bag. How many small bags can he make based on the following situations?

	Situation	Think about it (what is the question asking)	Picture It	Write it in symbols	Process it (What do you actually do?)	Solve it
4.	$\frac{1}{5}$ pound					
5.	$\frac{1}{6}$ pound					
6.	$\frac{3}{4}$ pound					

(Taken from *TEXTEAMS Rethinking Middle School Mathematics: Numerical Reasoning*)

Mr. Mangham is making ribbons for all of his students to wear in honor of his former student, Marci Holden. It takes  $\frac{1}{6}$  of a yard to make a ribbon for each student. How many badge ribbons can he make from the lengths listed below? For each answer that has a remainder, some ribbon left over, tell what fractional part of another badge ribbon you could make with the amount left over.

On a piece of notebook paper or computer paper create the six columns shown below. You will answer the following questions by completing the row for each question. The first one has been done for you.

	<b>Situation</b>	<b>Think about it (what is the question asking)</b>	<b>Picture It</b>	<b>Write it in symbols</b>	<b>Process it (What do you actually do?)</b>	<b>Solve it</b>
1.	$\frac{1}{2}$ yard	$\frac{1}{2}$ yd. divided into pieces of $\frac{1}{6}$ yd.		$\frac{1}{2} \div \frac{1}{6}$	Divide yard in half, whole into sixths. How many sixths in a half?	3
2.	$\frac{3}{4}$ yard					
3.	$\frac{5}{8}$ yard					
4.	$2\frac{2}{3}$ yard					

Next, Mr. Mangham is making bows for the math students that are not in his classes so that he can easily recognize them. It takes  $\frac{1}{2}$  yard of ribbon to make one bow. How many bows can Mr. Mangham make from each of the following amounts of ribbon?

	<b>Situation</b>	<b>Think about it (what is the question asking)</b>	<b>Picture It</b>	<b>Write it in symbols</b>	<b>Process it (What do you actually do?)</b>	<b>Solve it</b>
5.	$\frac{4}{5}$ yard					
6.	$\frac{8}{9}$ yard					
7.	$1\frac{3}{4}$ yard					
8.	$2\frac{1}{3}$ yard					

Avery's father donated 6 gallons of paint to her school. The teachers at Avery's school have decided to paint tabletops with the paint. It will take  $\frac{1}{5}$  of a gallon of paint to cover a small-sized table. It will take  $\frac{1}{4}$  of a gallon of paint to cover a medium-sized table, and  $\frac{1}{3}$  of a gallon of paint for a large-sized table.

**Solve. Show all work on a separate sheet of paper.**

1.	How many small-sized tables can the teachers paint if they use all of the paint? Show a model and write an expression that relates to your model.
2.	How many medium-sized tables can the teachers paint if they use all of the paint? Show a model and write an expression that relates to your model.
3.	How many large-sized tables can the teachers paint if they use all of the paint? Show a model and write an expression that relates to your model.
4.	Based on your observations, write a rule that makes sense for dividing any whole number by a unit fraction.

Avery found some green tables and blue tables that needed painting in the cafeteria. Her father agreed to donate 6 more gallons of paint. It will take  $\frac{2}{3}$  of a gallon to repaint the green tables and  $\frac{4}{5}$  of a gallon to repaint the blue tables.

**Solve. Show all work on a separate sheet of paper.**

5.	If the teachers use all the paint, how many green tables can they paint? Use a model and write an expression that relates to the model.
6.	If the teachers use all the paint, how many blue tables can they paint? Use a model and write an expression that relates to the model.
7.	In question #4 you described a method for dividing fractions. Does your method work for questions #5 and #6? If needed, revise your rule so that it does work.

8. Use your rule to solve the following problems. What do you notice about your answers?

$4 \div \frac{1}{12}$	$4 \div \frac{2}{12}$	$4 \div \frac{4}{12}$
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9.	Based on your answers above can you predict the answer to $4 \div \frac{8}{12}$ ? Solve the problem. Was your prediction correct?
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$\frac{2}{3} \div \frac{5}{4} = \frac{2}{3} \times \frac{4}{5} = \frac{2}{3} \times \frac{4}{5} = \frac{2}{3} \times \frac{4}{5} = \frac{2}{3} \times \frac{4}{5}$ $\frac{2}{3} \times \frac{4}{5} = \frac{2 \times 4}{3 \times 5} = \frac{8}{15}$	<p><b>Any number times its reciprocal equals 1.</b> The reciprocal of <math>\frac{4}{5}</math> is <math>\frac{5}{4}</math>.</p> <p>So we need to multiply both the numerator and denominator by <math>\frac{5}{4}</math>.</p> <p><b>To divide fractions, multiply the first fraction by the reciprocal of the second number.</b></p>
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**Reciprocal** comes from “re” meaning backward and “pro” meaning forward. In writing the reciprocal you have gone “back and forth” and returned to the identity for multiplication.

Find the reciprocal of each number.

1.	$\frac{1}{4}$		2.	$\frac{5}{6}$		3.	7		4.	$\frac{8}{15}$	
5.	$8\frac{3}{4}$		6.	$9\frac{6}{7}$		7.	$7\frac{5}{6}$		8.	$3\frac{5}{12}$	

Find each quotient. Show all work on a separate sheet of paper. Simplify, but improper fractions are okay.

9.	$8 \div \frac{4}{5}$	10.	$\frac{1}{6} \div \frac{2}{9}$	11.	$\frac{9}{10} \div \frac{1}{4}$
12.	$\frac{3}{8} \div 9$	13.	$\frac{8}{9} \div \frac{2}{3}$	14.	$\frac{2}{5} \div \frac{4}{7}$
15.	$15 \div \frac{5}{9}$	16.	$\frac{7}{8} \div \frac{7}{10}$	17.	$\frac{1}{9} \div \frac{5}{12}$

Solve each equation. Show all work on a separate sheet of paper.

18.	$j = \frac{6}{7} \div \frac{3}{14}$	19.	$\frac{4}{9} \div \frac{14}{15} = b$	20.	$\frac{9}{16} \div \frac{3}{4} = s$
21.	$\frac{5}{12} \div 2\frac{1}{2} = p$	22.	$s = 2\frac{2}{3} \div 1\frac{5}{6}$	23.	$a = 1\frac{4}{5} \div 6$

Name the reciprocal (also called the multiplicative inverse) for each rational number.

1.	$-3\frac{3}{4}$		2.	-10		3.	$-\frac{8}{7}$		4.	0.6	
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**You may leave your answer as a simplified improper fraction for all problems below.**

Solve each equation. Show all work on a separate sheet of paper.

5.	$2\frac{1}{2} \div 3\frac{3}{5} = k$	6.	$\frac{4}{9} \div -6 = b$
7.	$c = 2\frac{1}{5} \div \left(-1\frac{7}{10}\right)$	8.	$-1\frac{1}{9} \div 1\frac{17}{63} = d$
9.	$-10 \div \left(-5\frac{3}{4}\right) = k$	10.	$g = -\frac{1}{5} \div \frac{7}{8}$
11.	$-12\frac{1}{4} \div 4\frac{2}{3} = x$	12.	$-5\frac{2}{7} \div \left(-\frac{3}{8}\right) = r$
13.	$-7\frac{3}{5} \div \left(-1\frac{9}{10}\right) = y$	14.	$6\frac{2}{3} \div \left(-\frac{10}{3}\right) = p$

15.	Reg Morris holds the world's crawling record (true story). He crawled $28\frac{1}{2}$ miles on a measured course $1\frac{1}{2}$ miles long. To set the record, he crawled without stopping for $9\frac{1}{2}$ hours. How many laps did Morris complete? What was his crawling speed in mi/h?
16.	A turtle walked $\frac{1}{2}$ mile at the rate of $\frac{1}{5}$ mile per hour. How long did it take?
17.	A certain math book is $\frac{3}{4}$ of an inch thick. How many of these books will fit on a shelf that is 3 feet wide?
18.	How many complete games of chess can Yuri play in 1 hour if playing 18 games takes him $3\frac{3}{10}$ hours?

Evaluate each expression. Show all work on a separate sheet of paper.

19.	$y \div z$ , if $y = \frac{4}{5}$ and $z = \frac{2}{3}$	20.	$c \div d$ , if $c = 14$ and $d = \frac{7}{8}$
21.	$f \div g$ , if $f = 5\frac{1}{4}$ and $g = 1\frac{5}{9}$	22.	$w \div z$ , if $w = 9$ and $z = 2\frac{1}{7}$

**Write an expression for each word problem and solve on a separate sheet of paper.**

1.	Draw a diagram to show how many $\frac{3}{4}$ -ft. pieces of string can be cut from a piece of string four and a half feet long.
2.	How many $\frac{3}{4}$ -c servings are there in a 6 c package of rice?
3.	George cut 5 oranges into quarters. How many pieces of orange did he have?
4.	Anna bought a package of ribbon 10 yd long. She needs $1\frac{1}{3}$ -yd pieces for a bulletin board. How many pieces can Anna cut from the ribbon?
5.	Using #4, what if Anna decided to use $\frac{2}{3}$ -yd pieces? How many pieces can she now cut?
6.	A bulletin board is 56 in. wide and 36 in. high. How many $3\frac{1}{2}$ -in columns can be created?
7.	There are 3 boys and 2 girls in the Krunch family. Mr. Krunch bought $3\frac{1}{2}$ pounds of candy to divide equally among them. How much candy did each child get?
8.	It takes 1 cup of liquid fertilizer to make $7\frac{1}{2}$ gallons of spray. How much liquid fertilizer is needed to make 80 gallons of spray?
9.	Darlene has $2\frac{1}{4}$ hours to complete three household chores. If she divides her time evenly, how many hours can she give to each?
10.	Elizabeth bought $3\frac{1}{3}$ pounds of tomatoes for $2\frac{1}{2}$ dollars. How much did she pay per pound?
11.	Dad paid \$2.00 for a $\frac{3}{4}$ pound box of candy. How much is that per pound?
12.	The runner ran $2\frac{1}{2}$ miles in $\frac{3}{4}$ hour. At that rate, how many miles could he run in 1 hour? That is, what is his speed in miles per hour?
13.	Farmer Brown measured his remaining insecticide and found that he had two and a quarter gallons. It takes three-fourths gallon to make a tank of mix. How many tankfuls can he make?
14.	Linda has $4\frac{2}{3}$ yards of material. She is making baby clothes for her niece. If each pattern requires $1\frac{1}{6}$ yards of material, how many patterns will she be able to make?
15.	On Sunday, Mr. Underwood swam $15\frac{2}{5}$ miles in 2.1 hours. As a mixed number, what was Mr. Underwood's swimming pace in miles per hour?

Determine the missing dimension of each of the following rectangles. All area units are in square inches.

$$A = lw$$

1.	$A = 32\frac{1}{2}in^2$	Length = $6\frac{1}{2}$ inches	2.	$A = 38\frac{1}{2}in^2$	Length = $5\frac{1}{2}$ inches
3.	$A = 42\frac{3}{4}in^2$	Width = $9\frac{1}{2}$ inches	4.	$A = 72\frac{1}{4}in^2$	Width = $8\frac{1}{2}$ inches
5.	$A = 117\frac{7}{8}in^2$	Length = $10\frac{1}{4}$ inches	6.	$A = 120\frac{3}{8}in^2$	Length = $4\frac{1}{2}$ inches

1.	$P =$	1.	$A =$
2.	$P =$	2.	$A =$
3.	$P =$	3.	$A =$
4.	$P =$	4.	$A =$
5.	$P =$	5.	$A =$
6.	$P =$	6.	$A =$

7.	Joni is making cookies for a picnic. The recipe calls for $2\frac{1}{3}$ cups of flour for a recipe that makes 12 cookies. She needs to make 120 cookies. How many batches will she have to make to get 120 cookies? How many cups of flour will she need in order to make 120 cookies?
8.	Haley bought $\frac{2}{3}$ pound of walnuts which cost \$4.25 per pound, $\frac{1}{3}$ pound of pecans which cost \$5.69 per pound, $\frac{3}{4}$ pound of peanuts at \$2.99 per pound and $\frac{1}{2}$ pound of almonds at \$3.98 per pound. How many pounds of nuts did Haley buy?
9.	For the problem above how much was the total cost of the nuts she bought?
10.	If each loaf of banana bread weighs $1\frac{3}{4}$ pounds, what is the total weight of 6 loaves of bread?
11.	Brad is making cookies for his Boy Scout troop. He needs 116 cookies. He can bake a dozen cookies every $12\frac{1}{2}$ minutes. How long will it take him to bake 116 cookies

You and your friends are planning a camping trip. You are in charge of making and bringing enough trail mix. The trail mix needs to provide each person with one serving for each day of the trip. Here is the recipe for trail mix:

$2\frac{1}{4}$ cups raisins	$\frac{1}{3}$ cup walnuts	$\frac{1}{8}$ cups coconut pieces
$\frac{3}{4}$ cup peanuts	$\frac{1}{8}$ cup cashew nuts	$\frac{1}{4}$ cup chocolate chips
$\frac{2}{3}$ cup dried pineapple pieces	<b>Combine all ingredients and mix them.</b>	

1.	How many cups in all does the recipe make?				
2.	If one serving is $\frac{1}{2}$ cup, how many servings does the recipe make?				
3.	Suppose 12 people go camping for 3 days. How many cups in all should you make?				
4.	For #3, how much of each ingredient should you use to make enough trail mix for each person to have a $\frac{1}{2}$ -cup serving on each day?	Raisins		Cashews	
		Peanuts		Coconuts	
		Pineapple		Chocolate	
		Walnuts			
5.	Suppose 3 people are going on the camping trip for 3 days and each serving of trail mix is $\frac{1}{4}$ cup. How many cups in all should you make?				
6.	For #5, how much of each ingredient should you use?	Raisins		Cashews	
		Peanuts		Coconuts	
		Pineapple		Chocolate	
		Walnuts			

**Write an expression for each word problem and solve on a separate sheet of paper.**

1.	Liz Ann and Bryan are running a race. Bryan only ran $1\frac{1}{4}$ miles before he dropped out. Liz Ann ran $3\frac{1}{2}$ times farther than Bryan. How far did Liz Ann run and how many more miles did Liz Ann run than Bryan?
2.	George Bush was eating his daily jelly beans. He decided he would share his jar of $5\frac{2}{5}$ pounds of jelly beans amongst himself and seven other world leaders. How many pounds did each leader get?
3.	Trent bought $6\frac{1}{2}$ pounds of tomatoes. An average tomato weighs $\frac{3}{8}$ of a pound. Approximately how many tomatoes did Trent buy?
4.	The Honey Baked Ham Store is busy during the holiday season. The average ham they sell weighs $9\frac{1}{2}$ pounds. If they say each ham will make $23\frac{1}{4}$ servings, how much does one serving weigh?
5.	Bob is very interested in knowing exactly how fast he drives. On his last trip, he figured he drove $42\frac{1}{4}$ miles in $1\frac{6}{7}$ hours. How fast was he driving?
6.	Bill owns a bakery that makes really good sugar cookies. It takes $12\frac{1}{5}$ cups of sugar to bake 5 complete batches and $\frac{1}{3}$ of another batch. How many cups of sugar is used for each batch?
7.	Garrett can make $3\frac{1}{2}$ pies in an hour. If he works for $7\frac{3}{4}$ hours, how many pies can Garrett make?
8.	In a recipe you need $4\frac{1}{2}$ cups of milk and you want to use all 15 cups of milk in the refrigerator. How many batches of the recipe can you make?
9.	Larry, who lives in London, won the equivalent of $2\frac{1}{2}$ million dollars in the lottery. If one dollar is equal to $1\frac{1}{2}$ pounds (money in England), how many pounds did he win?
10.	Becky has $3\frac{1}{5}$ pizzas left over from a party. The next day each person who visits Becky's house eats $\frac{2}{5}$ of a pizza. How many people visit before the pizza is all gone?
11.	Lizzy Lou had $\frac{3}{4}$ of a pumpkin pie. She decided to split it between herself, Emily Sue, Billy Bob, Joe Jill, and Danny Dude. How much of a whole pie will each person get?

**Write an expression for each word problem and solve on a separate sheet of paper.**

1.	Which expression does not have a value of $-0.75$ ?
	A $-\frac{4}{5} \div 0.6$ B $1.8 \div -2\frac{2}{5}$ C $\frac{15}{16} \div -1.25$ D $-0.9 \div 1\frac{1}{5}$
2.	Dorothy gives the munchkins $9\frac{1}{5}$ chocolate bars for Christmas. If there are 6 munchkins, how many chocolate bars does each one get?
3.	There were 10 cakes at Johnny's birthday party. If each person was given $\frac{2}{5}$ of a cake and all 10 cakes were eaten, how many people attended the party?
4.	There was $\frac{1}{2}$ of a pizza at Joe's Pizza, Pasta, & Subs. If each person at the restaurant was given $\frac{1}{16}$ of a whole pizza for free, how many people were at Joe's?
5.	The Griswold's sat down for Christmas Eve dinner. They bought a 24 pound turkey, which amazingly enough they cooked correctly this year. There are 6 people in their family so how many pounds does each person get? If Rusty eats only one-half of a pound and they split up the rest for the other five people, how much extra turkey will each get?
6.	If Emily practices band for $3\frac{2}{5}$ hours over 4 days, how many hours did she average each day?
7.	Santa eats $\frac{1}{4}$ of a cookie because he is on a diet. The rest of the cookie was divided up amongst all of the seven reindeer. Then Comet stole Blitzen's serving. How much of a cookie does Comet get?
8.	Taylor has $\frac{5}{7}$ of a pound of turkey and she has to make equal groups of $\frac{1}{3}$ of a pound each. How many groups are there?
9.	You are on a $9\frac{1}{3}$ mile hike with $6\frac{2}{3}$ gallons of water. How many gallons of water can you drink every mile?
10.	Scooby has $4\frac{1}{3}$ cups of dog food and needs to divide it up into serving sizes of $\frac{2}{3}$ of a cup each. How many servings can Scooby make?
11.	While catering a party, Lori spent $\frac{1}{4}$ of the time shopping, $\frac{1}{3}$ of the time cooking, and 2 hours cleaning up. How long did she work?

## **THINK YOU ARE HAVING A BAD DAY?**

Dear Sir:

I am writing in response to your request for additional information. In Block #3 of the accident reporting form, I put "trying to do the job alone" as the cause of my accident. You said in your letter that I should explain more fully and I trust that the following details will be sufficient.

I am a bricklayer by trade. On the day of the accident, I was working alone on the roof of a new six story building. When I completed my work, I discovered that I had about 500 pounds of bricks left over. Rather than carry the bricks down by hand, I decided to lower them in a barrel by using a pulley which, fortunately, was attached to the side of the building on the top floor.

Securing the rope at ground level, I went up to the roof, swung the barrel over the side and loaded bricks into it. Then I went back to the ground and untied the rope holding it tightly to insure a slow descent of the 500 pounds of bricks. You will note in Block #11 of the accident report form that my weight is 185 pounds. Do to my surprise at being jerked off the ground so suddenly, I lost presence of mind and forgot to let go of the rope. Needless to say, I proceeded at a rather rapid rate up the side of the building.

In the vicinity of the third floor, I met the barrel coming down. This explains my fractured skull, minor abrasions, and broken collarbone.

Slowed only slightly, I continued my rapid ascent, not stopping until the fingers of my right hand were two knuckles deep into the pulley. Fortunately by this time I had regained my presence of mind and was able to hold tightly to the rope despite the excruciating pain I was beginning to experience.

At that time however, the barrel of bricks reached the ground - and the bottom fell out of the barrel when it hit. Now devoid of the 500 pounds of bricks the barrel now weighed only 50 pounds.

As you might imagine, I began a rapid descent down the side of the building. In the vicinity of the third floor, I met the barrel coming up. This accounts for the two fractured ankles, broken tooth, and the severe lacerations on my legs and lower body.

Here my luck began to change slightly. The encounter with the barrel seemed to slow it up enough to lessen my injuries when I fell into the pile of bricks and fortunately only three vertebrae were cracked.

I am sorry to report, however, that as I lay there on the pile of bricks in pain, unable to move and watching the empty barrel six stories above me, I lost hold of the rope. The empty barrel weighed more than the rope and came back down on me and broke both my legs.

I hope I have furnished the information you required as to how the accident occurred.