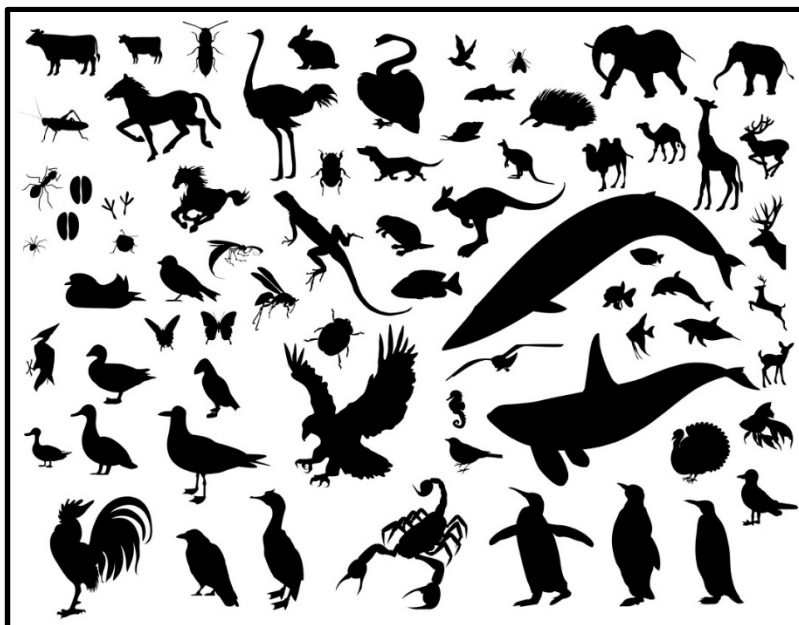




# *Animals Big and Small: Skin and Guts*

*What if elephants had small ears?*

A VOLUME AND SURFACE AREA PROJECT

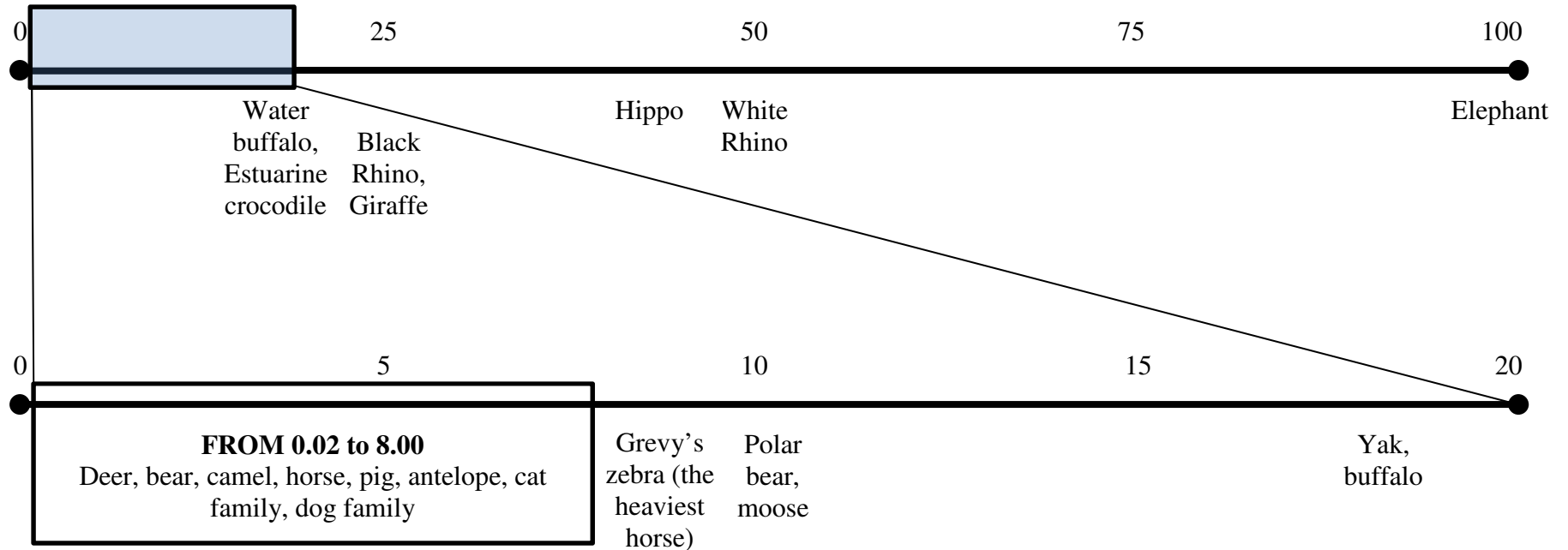


### Animal Definitions

The following are definitions which fit the **vast majority** of animals. There are some exceptions to each of the rules.

		<b>Species</b>	<b>Examples</b>
<b>Mammals</b>	Air-breathing animals, produce their own internal heat (warm-blooded), mothers nurse young with milk, have teeth, give birth to live young, has hair/fur	4,000	humans, dogs, cats, cows, elephants, pigs, whales, dolphins, horses
<b>Amphibians</b>	Lay eggs, do not produce their own internal heat (cold-blooded), lives on land, breeds in the water, smooth and moist skin	4,325	Frogs, toads, salamanders
<b>Reptiles</b>	Do not produce their own heat (cold-blooded), have scales	6,900	snakes, alligators, crocodiles, turtles, lizards
<b>Birds</b>	Have feathers, have wings, produce their own heat (warm-blooded), egg-laying, have beak, no teeth	9,700	Pigeons, hummingbirds, flamingos, parrots, bluejay, dove, duck
<b>Fish</b>	Lives wholly in the water, gills and fins, do not produce their own heat (cold-blooded), scales	45,000	Salmon, bass, perch, cod, goldfish, tuna
<b>Tetrapods</b>	Reptiles, birds, amphibians, and mammals		

# THE RELATIVE SIZE OF ANIMALS



- 0.002 to 0.06** most birds, lizards, frogs, toads, snakes
- 0.004 to 0.002** rats and mice
- 0.0004** hummingbirds
- 0.0003** bats and shrews
- 0 to 0.00004** *99.9% of all animals* – insects, arachnids, worms, crustaceans, etc.

NOTE: If the blue whales were placed on the number line, they would be at 2000.

**PART 1: Learning About Surface Area and Volume**

*You may use centimeter cubes and a calculator for this project.*

1. Create two figures for each number of cubes indicated. Make one figure represent the maximum surface area for that number of cubes and the second figure represent the minimum surface area.

Number of cubes	Maximum Surface Area (square units)	Minimum Surface Area (square units)
6		
7		
8		
9		
10		
$n$		no pattern

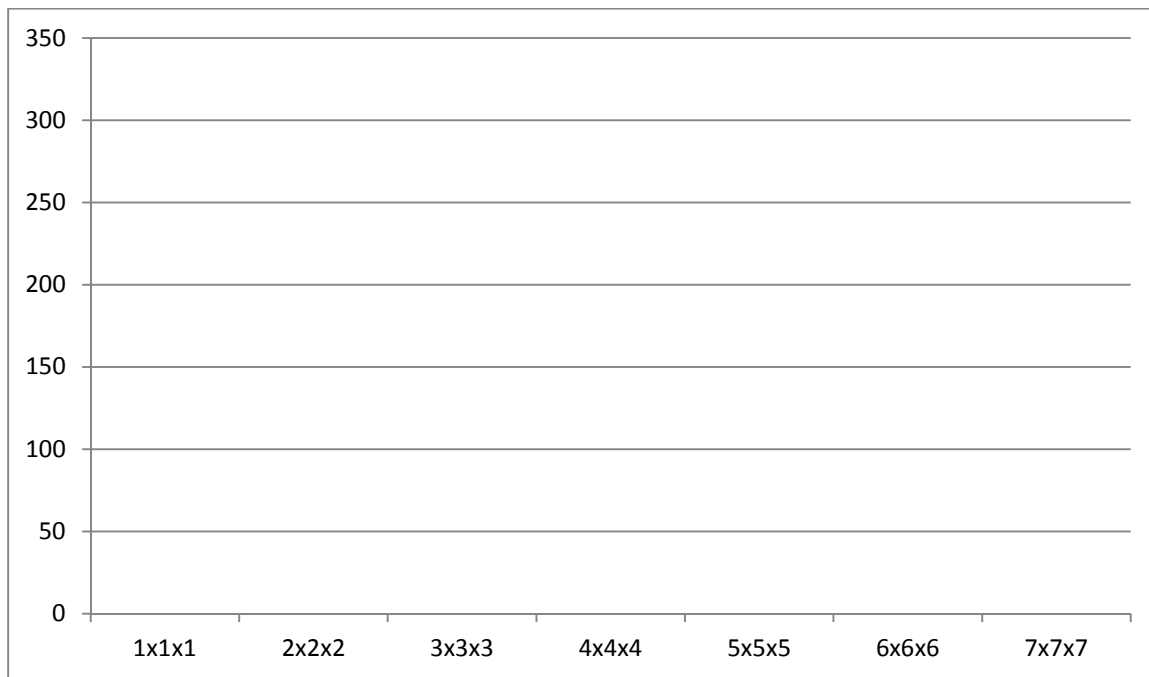
Think about what kind of shapes make the most and least surface area.

2. Complete the following table. Record the surface area and volume of each cube or shape.

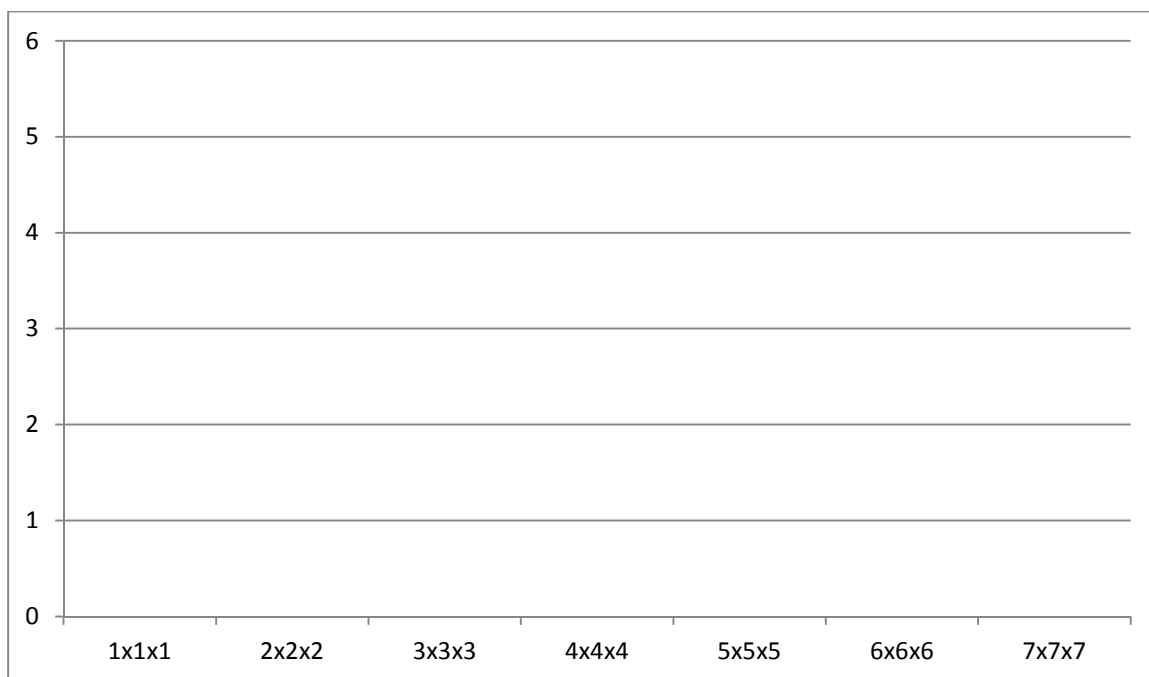
Edge of cube (units)	Dimensions (units)	Surface Area (square units)	Volume (cubic units)	Surface Area-to-Volume Ratio (simplified per unit)
1	1 x 1 x 1			6 to 1
2	2 x 2 x 2			to 1
3	3 x 3 x 3			to 1
4	4 x 4 x 4			to 1
5	5 x 5 x 5			to 1
6	6 x 6 x 6			to 1
7	7 x 7 x 7			to 1

3.	As you continue to increase the edge size of the cube, which will grow faster: surface area or volume?	
4.	What happens to the surface area-to-volume ratio as the cubes get larger?	
5.	When the edge of the cube doubles what happens to the surface area?	
6.	When the edge of the cube doubles what happens to the volume?	

7. On the graph below first plot the surface area for each of the seven cubes you examined in question 2 on the previous page. Create a line graph. Note that your graph will not be proportional or linear.
8. Then on the same graph plot the volume for each of the seven cubes you examined in question 2 on the previous page. Create a second line graph. Note that your graph will not be proportional or linear.



9. On the graph below plot the surface area-to-volume ratio for each of the seven cubes you examined in question 2 on the previous page. Create a line graph.



10. Using what you learned in the previous problems, complete the table below.

When the edge of the cube...	The surface area gets multiplied by...	And the volume gets multiplied by...
doubles (x2)		
triples (x3)		
quadruples (x4)		
goes up $m$ times		

11.	You have $3 \times 3 \times 3$ cube and a $7 \times 7 \times 7$ cube. What is the ratio of their surface areas? Use your tables above to help.	
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### **PART 2: Applying the Surface Area-to-Volume Ratio to Animals**

Why are flying squirrels in the Arctic more than 50% larger than those in Central America?

Animals adapt to their environment. Part of this adaptation involves both an animal's surface area and an animal's volume. How the surface area and volume compare can tell us a lot about the different places where animals live.

The surface-area-to-volume ratio is also called the surface-to-volume ratio.

**Animals generate heat internally in proportion to their volume.**

The larger the volume of the animal the more heat it can produce.

**Animals lose heat externally in proportion to their surface area.**

The larger the surface area of the animal the more heat it can lose.

Body temperatures of animals are usually greater than the outside temperature meaning that frequently the direction of heat 'flow' is from the animal to the outside, i.e. heat is lost from the animal. For a mammal heat lost to the outside, via the surface, must be replaced by heat obtained from the breakdown of food.

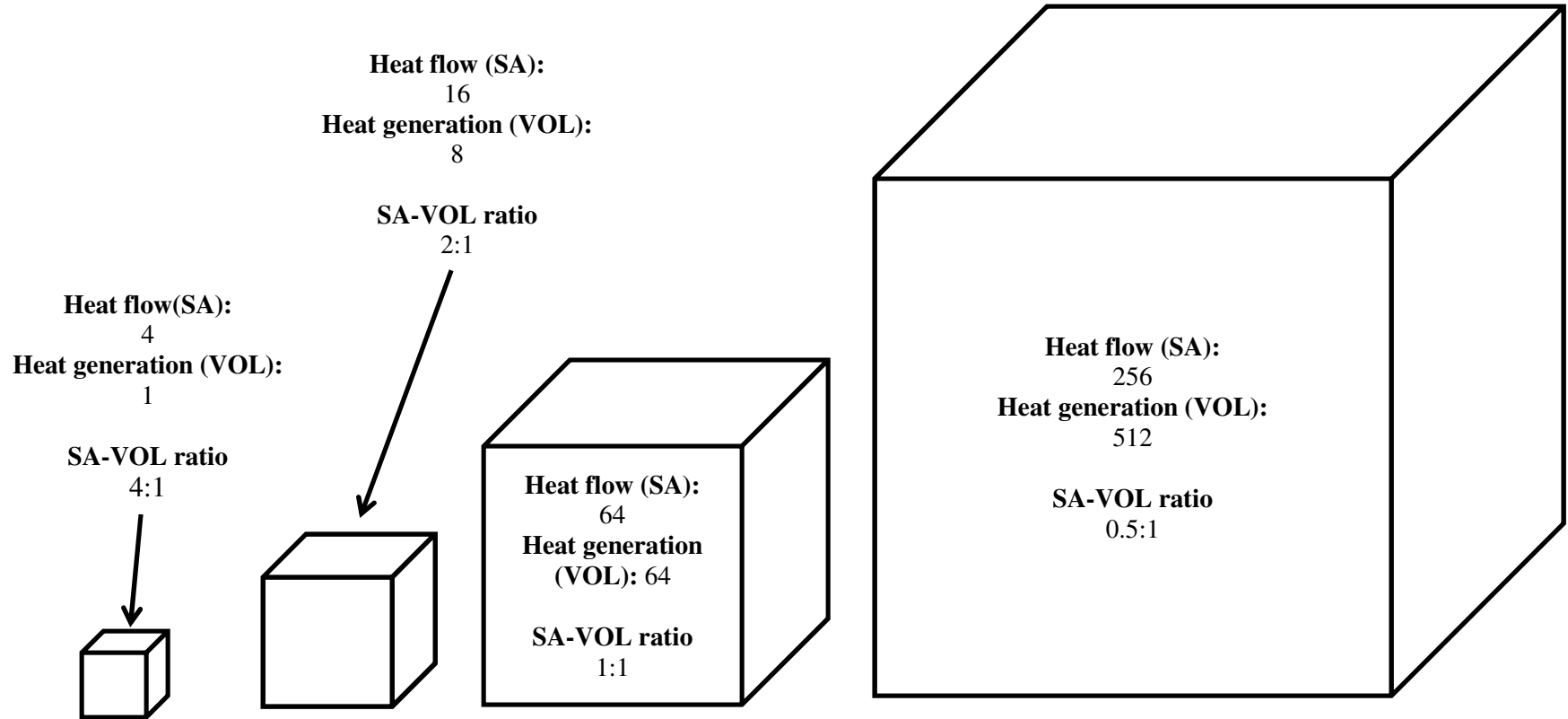
**The greater the surface area-to-volume ratio of an animal,  
the more heat it loses relative to its volume.**

As animals grow in size their inside (volume) gets "more bigger" than their outside (surface area). You proved this in part one when you completed table number two. As you increased the side length, the volume started growing much faster than the surface area.

The larger the animal, the smaller the surface area-to-volume ratio and so the less relative area there is to lose heat. This means that for identically shaped animals of different sizes, the large one will keep its temperature more easily. **Being bigger means being warmer.**

## Surface Area and Volume Comparison of Small and Large Animals

[The surface area and volume numbers are just for comparison purposes.]



We are small animals. We don't generate much heat and we don't have much heat flow. Compared to big animals, though, we can lose our heat much more easily and we can have a hard time staying warm.

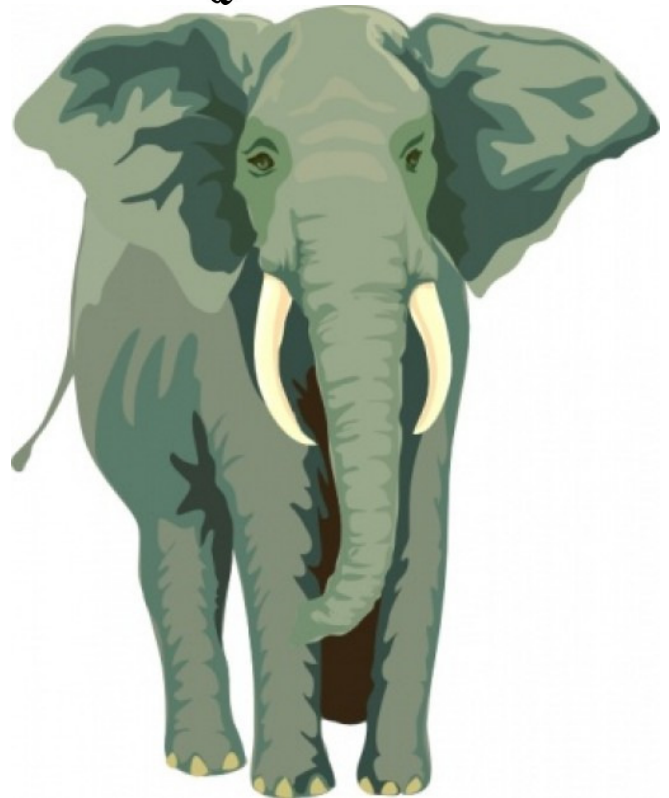
Smaller

Bigger

We are big animals. We generate a lot of heat and we have a large flow of heat. Compared to small animals, though, we have a hard time losing heat which means we stay warm much more easily.

12. Determine the surface area-to-volume ratio of the animals listed below.

Animal	Estimated surface area	Estimated volume	Estimated Surface Area-to-Volume Ratio (nearest hundredth)
Mouse	6 square inches	1 cubic inches	to 1
Rat	24 square inches	8 cubic inches	to 1
Lemming	40 square inches	16 cubic inches	to 1
Labrador Retriever	3,532 square inches	13,824 cubic inches	to 1
Zebra	5,760 square inches	27,648 cubic inches	to 1
Polar Bear	14,400 square inches	96,768 cubic inches	to 1
Elephant	36,000 square inches	432,000 cubic inches	to 1
As the animal gets larger the surface area-to-volume ratio gets....			





**Animals generate heat internally in proportion to their volume.**

The larger the volume of the animal the more heat it can produce.

**Animals lose heat externally in proportion to their surface area.**

The larger the surface area of the animal the more heat it can lose.

13.	Which animal in #12 will generate the most heat? least heat?		
14.	Which animal in #12 will lose the most heat? least heat?		

As animals grow in size their inside (volume) gets “more bigger” than their outside (surface area). You proved this in part one when you completed table number two. As you increased the side length, the volume started growing much faster than the surface area.

The larger the animal, the smaller the surface area-to-volume ratio and so the less relative area there is to lose heat. This means that for identically shaped animals of different sizes, the large one will keep its temperature more easily. Being bigger means being warmer.

15.	Which animal in #12 has the biggest SA-VOL ratio and thus will lose the most heat relative to its size?	
16.	Which animal in #12 has the smallest SA-VOL ratio and thus will lose the least heat relative to its size?	

17.	Which characteristic can animals change the easiest and fastest: their surface area or their volume? Explain.	
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You will now use your surface area and volume knowledge and apply it to a wide variety of animals.

18.	Would you expect large desert animals to try to minimize or maximize their surface area-to-volume ratios? Why?	
19.	Would you expect large arctic animals to try to minimize or maximize their surface area-to-volume ratios? Why?	

**ELEPHANTS**

An elephant has a small surface area compared to its volume. Therefore, it has a very small surface area-to-volume ratio. Since elephants lose heat to their surroundings more slowly, they can overheat easily.

20.	<b>In terms of surface area and/or volume</b> , why do you think some elephants, like the African elephant, have extremely large ears (the largest earflap in history of any animal)? Hint: It is not to swat away flies or make a fan.	
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**MICE**

A mouse has a very large surface area-to-volume ratio. It loses heat to its surroundings very quickly and must eat a lot of food to replace the energy lost. That huge metabolic rate makes for a high heart rate, and because mammalian hearts are only good for about one billion beats, that heat-replacing pace is what gives mice a live fast, die young lifestyle.

21.	Who will lose more total heat in a given period, a mouse or an elephant? Why?	
22.	Who will lose more heat relative to its volume, a mouse or an elephant? Why?	
23.	Who will need to eat the most food, a mouse or an elephant? Why?	
24.	Who will need to eat the most food relative to size, a mouse or an elephant? Why?	

**FOXES**

In general, similar animals have different ear sizes depending on the climate in which they live.

25.	The arctic fox has much smaller ears than the fennec fox, which lives in the desert. <b>In terms of SA-VOL ratio</b> , why?	
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**BEARS/POLAR BEARS**

Some bears can almost become spherical. A sphere has the smallest surface area-to-volume ratio of any shape. While a one unit cube has a ratio of 6:1, a one unit diameter sphere has a ratio of only 4.84:1. The polar bear has stocky limbs and very short ears.

26.	<b>In terms of SA-VOL ratio</b> , why would a bear want to curl up into a ball (sphere)?	
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27.	Polar bears and camels are both very large animals. Why does a polar bear have short, stumpy legs while a camel has tall gangly legs?	
28.	A lemming (a small mouse-sized mammal of the tundra, related to a mole) is much smaller than a polar bear. <b>In terms of SA-VOL ratio</b> , which species loses body heat more easily?	
29.	<b>In terms of surface area and/or volume</b> , why does a polar bear have very short ears?	

**PEOPLE**

30.	<b>In terms of surface area and/or volume</b> , why do people fold their arms over their chest or even sit hugging their knees to their chin when they are cold?	
31.	Who will lose more total heat in a given period, an infant or an adult?	
32.	Who will lose more heat relative to its volume, an infant or an adult?	
33.	Who will need to eat the most food, an infant or an adult?	
34.	Who will need to eat the most food relative to size, an infant or an adult?	

**BIRDS**

35.	<b>In terms of SA-VOL ratio</b> , why are birds in the Antarctic (the Emperor penguin, for example) very large compared to many birds found other places?	
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**SMALLER and LARGER**

White-tailed deer are larger in Canada than in the Florida Keys. The common wolf is 20% larger in northern Canada than in northern Mexico. Mountain lions are smaller closer to the equator. Flying squirrels in the Arctic are more than 50% larger than those in Central America.

36.	<b>In terms of SA-VOL ratio</b> , why are all these animals larger in some places and smaller in other places?	
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**WARM-BLOODED and COLD-BLOODED**

*Warm-blooded animals* try to maintain a constant body temperature. They generate their own heat and they cool themselves. Warm-blooded animals can shiver - the most effective way of producing heat.

*Cold-blooded animals* take on the temperature of their surroundings. They are more active in warm environments and sluggish in cold environments. Cold-blooded animals cannot shiver.

37.	Small warm-blooded animals have a large surface area-to-volume ratio. What does this tell you about the amount of food they will need to eat?	
38.	Where would you expect warm-blooded animals to have a slender body design? Where would you expect warm-blooded animals to have a chubby body design?	
39.	Why does having a large surface area-to-volume ratio help small cold-blooded creatures warm up when they need to?	

**LIZARDS/REPTILES**

Just about all reptiles are cold-blooded. Lizards are cold-blooded.

40.	<b>In terms of surface area and/or volume</b> , why does a lizard stretch out to bask in the sun in the morning?	
41.	<b>In terms of surface area and/or volume</b> , why are most reptiles long and slender?	
42.	Why do most reptiles only need to eat sparingly?	
43.	<b>In terms of SA-VOL ratio</b> , why do most land mammals, lizards, and snakes curl up in a cold environment?	

44.	An animal has a volume of 720 cubic inches. A smaller animal has dimensions that are half the size of the larger animal. What is the volume, in cubic inches, of the smaller animal?	
45.	How many 2 by 2 by 2 inch animals could fit inside a 4 by 4 by 4 inch box?	
46.	An animal in the general shape of a prism has volume of 1000 cm <sup>3</sup> . What is the smallest surface area possible for this animal? What is the smallest SA-VOL ratio possible for this animal?	

## Skin & Guts Final Project

Based on what you have learned about different animals, their volumes, and their surface areas, you now get the chance to create your own animal.

Design a new species of animal choosing between one of the two options below. All designs should be on computer paper or larger art pad paper. *As you decide on a design, make sure it is one that will demonstrate your knowledge of surface area and volume.*

<b>Design 1</b>	Create a new land tetrapod that would be ideally suited to live at the Earth's equator in areas that have the highest temperatures. Make your design creative and fun while at the same time using your knowledge of surface area-to-volume ratios.
<b>OR</b>	
<b>Design 2</b>	Create a new land tetrapod that would be ideally suited to live at one of the extremes of the Earth (Arctic or Antarctic) in areas that have the lowest temperatures. Make your design creative and fun while at the same time using your knowledge of surface area-to-volume ratios.
<b>Optional Extra Credit</b>	Create a 3D scale model of your animal. You choose the scale and you choose any materials you wish to create your scale model.

Your picture:

- Should have the animal's name
- Should show the animal length-wise [You may draw additional angles of your animal if you wish.]
- Should be similar to a picture you would find in an animal encyclopedia [draw some sort of background to give the reader an idea of its habitat.]
- Should have a background helping the reader to understand its environment
- Should list its dimensions
- Should list the total surface area, total volume, and the SA-VOL ratio of your animal
- Should provide a description of the animal and its habitat.
- Should include a description include what it eats (omnivore, herbivore, carnivore) and its predators and prey.
- Can optionally be colored
- Will be graded on how well you incorporate the math you have learned into its shape, size, characteristics, and dimensions

**Your project may be photographed and placed on ManghamMath when completed. Please ensure that it is neat and can easily be read. I suggest a final copy in Sharpie or marker to make it easier to see.**

## SUGGESTED LAYOUT

<p style="text-align: center;">ANIMAL DESCRIPTION &amp; INFORMATION</p>	<div style="border: 1px dashed black; padding: 5px; margin-bottom: 10px;"> <p><b>ANIMAL NAME</b> Lives in: _____ Most similar real-life animal(s): _____</p> </div> <div style="border: 2px dashed black; border-radius: 20px; background-color: #e0e0e0; padding: 20px; margin: 10px auto; width: 80%;"> <p style="text-align: center;">ANIMAL PICTURE  (best if drawn in its environment)</p> </div>	<p>DIMENSIONS <i>(l,w,h)</i> TOTAL SA TOTAL VOL SA:VOL RATIO</p> <p>Why is the ratio big or small? **</p>
<p style="text-align: center;">HABITAT DESCRIPTION</p>		<p style="text-align: center;">PREDATORS PREY ADDITIONAL INFORMATION</p>

**\*\* SA-VOL RATIO**

Small animals:	Greater than 1:1
Medium animals:	Between 0.5:1 and 1:1
Large animals:	Less than 0.5:1

**Determining the volume and surface area of your animal**

You may use a calculator. Use inches as all units of measurement.

If you have other important attributes (neck, shell, feet, wings, etc.) include them on the Additional Volume and Additional Surface Area lines.

**SHAPES USED**

Shape of head	
Shape of body	
Shape of legs	

**VOLUME**

Volume of head	
Volume of body	
Volume of 1 leg	
Volume of 4 legs	
Additional Volume not listed above	
<b>TOTAL VOLUME (in<sup>3</sup>)</b>	

**SURFACE AREA**

Surface Area of head	
Surface Area of body	
Surface Area of 1 leg	
Surface Area of 4 legs	
Additional Surface Area not listed above	
<b>TOTAL SURFACE AREA (in<sup>2</sup>)</b>	

<b>Surface Area to Volume Ratio</b> (nearest hundredth)	to 1
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Answer the questions below regarding your new animal. Most of these items should be on your layout.

1.	What is the name of your new animal?			
2.	Where does your animal live?			
3.	To what real-world animal would it be closest in size?			
4.	How tall is your animal? How long is your animal? How wide is your animal? (all in inches)			
5.	After you decided on the volume (size) of your animal, what characteristics did you add to either increase or decrease the overall surface area of your animal?			
6.	If your animal's volume was ten times bigger than your original design, how might you have to change some parts of its body to increase or decrease surface area?			
7.	If your animal's volume was one-tenth of the size of your original design, how might you have to change some parts of its body to increase or decrease surface area?			

8.	<p>You have two dogs, one that weighs 30 pounds and one that weighs 60 pounds. Which of the following best describes how the big dog eats compared to the small dog?</p> <p>1. The same amount    2. Less    3. More, but not twice as much          4. Twice as much    5. More than twice as much</p>	
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- Warm-blooded animals have a limited variety of shapes because large SA to VOL means large heat escape. Thus they are more spherical than other animals.
- Cold-blooded, like lizards and snakes, can be small and skinny.
- Ants can lift 10-50x their own weight.
- Rhinoceros beetles can lift 850x their own weight.
- One way for simple water creatures to become bigger is to get more skin for getting more oxygen (oxygen drifts in through skin) – make their skin “foldy” to increase its SA.
- Bigger water animals have found that it is better to have all your foldy skin for getting oxygen in one place – thus gills.
- On land gills would dry up so the land creatures developed lungs. They were simple bags at first and got more and more complicated – having bigger and bigger SA for getting more oxygen. Mammal lungs give a really huge area of thin skin for breathing.
- Commerson’s dolphin is one of the smallest mammals to live its whole life in the water and it is the size of a Labrador retriever. They eat a lot and have a thick layer of fat.
- Most water mammals are much bigger so they can hold onto their heat better.
- Being too small is bad for a reptile because they can lose heat quickly and become sluggish.
- Big animals can go longer without food than small animals. Thus they can migrate more.
- Whales migrate thousands of miles, but dolphins generally stay in one place.
- Wildebeests migrate 1800 miles per year, while smaller plant eaters do not.
- Emperor penguins survive at -76 degrees and 186 miles per hour wind. They weigh 66-88 pounds. They do this by huddling to decrease their SA.
- The deserts are home to many small animals that can hide during the heat of the day and also to many big animals that are able to stand exposed to the elements and tough it out.
- The deserts do not have many medium sized animals. Exception: jackrabbits. One-third of their heat can be dumped through their ears to remain cool.

<b>Animal</b>	<b>Food Eaten</b>	<b>Weight</b>	<b>Food Eaten:Weight Ratio</b>
Elephant	400 pounds a day	9000 pounds	0.04:1
Male polar bear	150 pounds in a feeding	1500 pounds	0.10:1
Tiger	77 pounds of meat in a feeding	500 pounds	0.15:1
Female hamster	0.4 ounces per day	3.5 ounces	0.11:1
Vampire bat	1 ounce per day	1 ounce	1.00:1
Hummingbird	0.07 ounces	0.11 ounces	0.64:1
Queen bee	0.32 ounces per day	0.004 ounces	80.00:1

**Linear Equations**

Slope-intercept form	$y = mx + b$	Direct Variation	$y = kx$ (8 <sup>th</sup> grade)
Constant of proportionality	$k = \frac{y}{x}$	Slope of a line	$m = \frac{y_2 - y_1}{x_2 - x_1}$ (8 <sup>th</sup> grade)

<b>Circumference</b>	Circle	$C = 2\pi r$ or $C = \pi d$
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**Area**

Rectangle	$A = bh$	Trapezoid	$A = \frac{1}{2}(b_1 + b_2)h$
Parallelogram	$A = bh$	Circle	$A = \pi r^2$
Triangle	$A = \frac{bh}{2}$ or $A = \frac{1}{2}bh$		

<b>Surface Area</b> (8 <sup>th</sup> grade)	<b>Lateral</b>	<b>Total</b>
Prism	$S = Ph$	$S = Ph + 2B$
Cylinder	$S = 2\pi rh$	$S = 2\pi r^2 + 2\pi rh$
Sphere		$S = 4\pi r^2$

**Volume**

Triangular prism	$V = Bh$	Cylinder	$V = Bh$ or $V = \pi r^2 h$ (8 <sup>th</sup> grade)
Rectangular prism	$V = Bh$	Cone	$V = \frac{1}{3}Bh$ or $V = \frac{1}{3}\pi r^2 h$ (8 <sup>th</sup> )
Pyramid	$V = \frac{1}{3}Bh$	Sphere	$V = \frac{4}{3}\pi r^3$ (8 <sup>th</sup> grade)

<b>Pi</b>	$\pi \approx 3.14$ or $\pi \approx \frac{22}{7}$
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<b>Distance</b>	$d = rt$	<b>Compound Interest</b>	$A = P(1 + r)^t$
<b>Simple Interest</b>	$I = prt$	<b>Pythagorean Theorem</b>	$a^2 + b^2 = c^2$ (8 <sup>th</sup> grade)