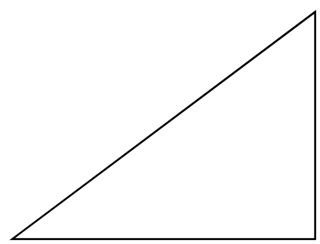
ACCELERATED MATHEMATICS

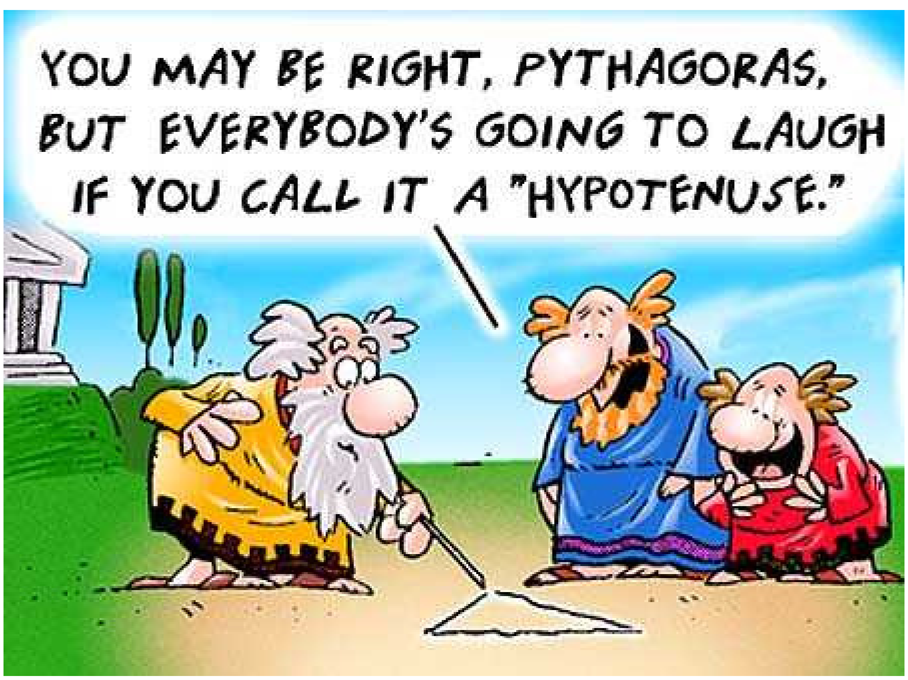
CHAPTER 14

<u>PYTHAGOREAN THEOREM</u>

TOPICS COVERED:

- Simplifying Radicals
- Pythagorean Theorem
- Distance formula





Created by Lance Mangham, 6th grade math, Carroll ISD

In this chapter, radicals are going to be simplified. To simplify means to find another expression with the exact same value. It does not mean to find a decimal approximation.

To simplify a radical use the following steps:
1. If the number under your radical cannot be divided evenly by any of the perfect squares, your radical is already in simplest form. Perfect squares are numbers such as 1, 4, 9, 16, 25, 36,…

2. Find the **largest** perfect square which will divide evenly into the number under your radical sign. This means that when you divide, you get no remainders, no decimals, no fractions.

Ex. $\sqrt{20}$ 4 is a perfect square that divides evenly into 20.

3. Write the number appearing under your radical as the product of the perfect square and your answer from dividing. Then give each number in the product its own radical sign and simplify.

$$\sqrt{20} = \sqrt{4 \bullet 5} = \sqrt{4} \bullet \sqrt{5}$$
$$\sqrt{4} \bullet \sqrt{5} = 2\sqrt{5}$$

Example #2: $\sqrt{72} = \sqrt{36 \bullet 2} = \sqrt{36} \bullet \sqrt{2} = 6\sqrt{2}$

If you get stuck thinking of a perfect square you can also draw a factor tree. For example, a tree for 300 would give you 3, 5, 5, 2, 2. So that is $\sqrt{3 \bullet 5 \bullet 5 \bullet 2 \bullet 2} = 5 \bullet 2\sqrt{3} = 10\sqrt{3}$

Simplify the following radicals.

1.	$\sqrt{18}$		2.	$\sqrt{48}$	
3.	$\sqrt{300}$		4.	$\sqrt{192}$	
5.	$\sqrt{75}$		6.	$\sqrt{144}$	
7.	$\sqrt{8}$		8.	$\sqrt{800}$	
9.	$\sqrt{242}$		10.	$\sqrt{80}$	
11.	$2\sqrt{50}$		12.	$\sqrt{180}$	
13.	$5\sqrt{98}$		14.	$\sqrt{40}$	
15.	$\sqrt{243}$		16.	$\sqrt{2000}$	
17.	$\sqrt{\frac{16}{25}}$		18.	$\sqrt{\frac{64}{121}}$	

Operations with Radical Expressions

Ex. 1 $2\sqrt{3}+6\sqrt{3}=8\sqrt{3}$

Think of the square root part like a variable. $2x+6x=8x$

Ex. 2 $7\sqrt{5}-4\sqrt{5}=3\sqrt{5}$

Ex. 3 $6\sqrt{7}+2\sqrt{5}=....$

These terms cannot be combined since they have different square roots.
Think of this as being similar to $6x+2y$ which cannot be simplified.

Ex. 4 $3\sqrt{7}\bullet 4=12\sqrt{7}$

The whole numbers can be combined together by multiplying.

Ex. 5 $\dfrac{8\sqrt{3}}{2}=4\sqrt{3}$

Ex. 6 $2\sqrt{3}\bullet 5\sqrt{11}=10\sqrt{33}$

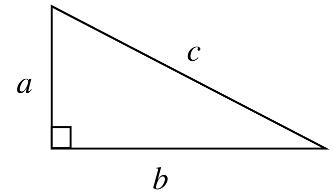
Simplify the following radicals.

1.	$3\sqrt{6}+7\sqrt{6}$		2.	$9\sqrt{13}+5\sqrt{13}$	
3.	$8\sqrt{2}+4\sqrt{2}$		4.	$15\sqrt{5}-8\sqrt{5}$	
5.	$7\sqrt{17}-2\sqrt{17}$		6.	$8\sqrt{4}-2\sqrt{4}$	
7.	$5\sqrt{7}+6\sqrt{5}$		8.	$7\sqrt{2}+4\sqrt{3}-5\sqrt{2}$	
9.	$5\bullet 6\sqrt{3}$		10.	$7(4\sqrt{5})$	
11.	$2\sqrt{37}\bullet 6$		12.	$\dfrac{15\sqrt{11}}{3}$	
13.	$\dfrac{24\sqrt{12}}{6}$		14.	$4\sqrt{5}\bullet 2\sqrt{4}$	
15.	$\left(6\sqrt{2}\right)^2$		16.	$7\sqrt{23}\bullet 2\sqrt{23}$	
17.	$\dfrac{5\sqrt{5}+11\sqrt{5}}{2}$		18.	$\dfrac{7}{3}\bullet\sqrt{\dfrac{9}{49}}$	

Activity 14-3: **Pythagorean Theorem** Name:

The Pythagorean Theorem: In a **right triangle**, the sum of the squares of the lengths of the **legs** is equal to the square of the length of the **hypotenuse**.

$$a^2 + b^2 = c^2$$

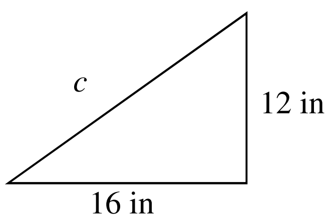


Example: A right triangle has legs of lengths 6 cm. and 8 cm. What is the length of the hypotenuse?

$$a^2 + b^2 = c^2$$
$$6^2 + 8^2 = c^2$$
$$36 + 64 = c^2$$
$$\sqrt{100} = \sqrt{c^2}$$
$$10 \text{ cm} = c$$

Write an equation you could use to find the length of the missing side of each right triangle. Then find the missing length. Simplify all radicals. Show all work on a separate sheet of paper.

1.



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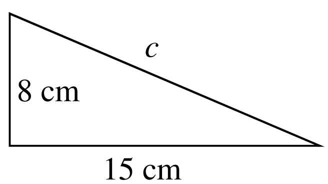

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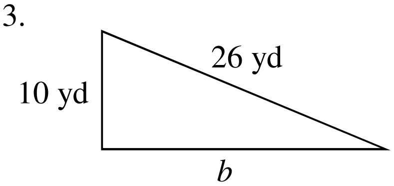

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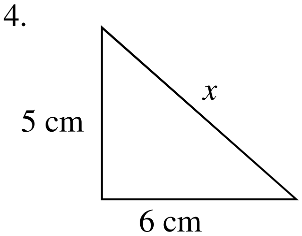

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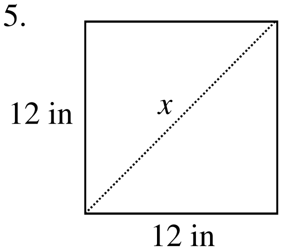

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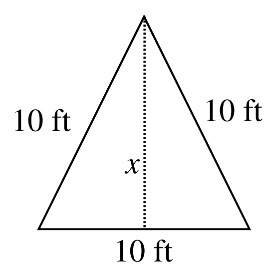


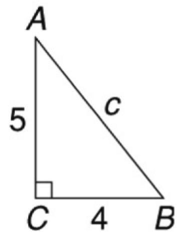
7.	a = 24 ft, b = 32 ft		8.	a = 9 ft, c = 16 ft		9.	b = 5 in, c = 11 in	

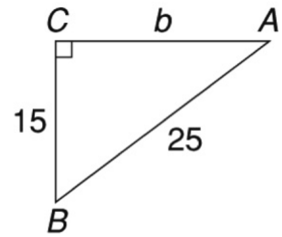
Determine whether each triangle with sides of given lengths is a right triangle.

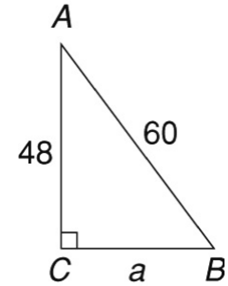
10.	6 cm, 8 cm, 10 cm		11.	9 mm, 12 mm, 16 mm		12.	18 ft, 80 ft, 82 ft	
13.	10 mi, 24 mi, 25 mi		14.	15 cm, 36 cm, 39 cm		15.	16 yd, 30 yd, 34 yd	

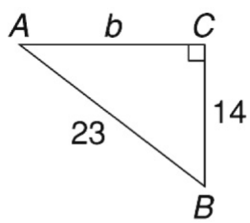
Activity 14-4: **The Pythagorean Theorem** Name:

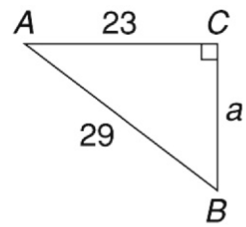
1-6. Find the missing side. Round to the nearest tenth, if necessary.

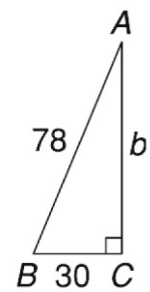












Solve.

7.	Jane and Miguel are siblings. They go to different schools. Jane walks 6 blocks east from home. Miguel walks 8 blocks north. How many blocks apart would the two schools be if you could walk straight from one school to the other?	
8.	The base of a rectangular box has a width of 3 inches and a length of 4 inches. The box is 12 inches tall. a. Draw a picture of the box. b. How far is it from one of the box's top corners to the opposite corner of the base of the box?	

Write "yes" for sides that form right triangles and "no" for sides that do not form right triangles. Prove that each answer is correct.

9. 7, 24, 25
10. 30, 40, 45
11. 10, 15, 18

Solve.

12.	A commuter airline files a new route between two cities that are 400 kilometers apart. One of the two cities is 200 kilometers from a third city. The other one of the two cities is 300 kilometers from the third city. Do the paths between the three cities form a right triangle? Prove that your answer is correct.	
13.	A school wants to build a rectangular playground that will have a diagonal length of 75 yards. How wide can the playground be if the length has to be 30 yards?	
14.	A 250-foot length of fence is placed around a three-sided animal pen. Two of the sides of the pen are 100 feet long each. Does the fence form a right triangle? Prove that your answer is correct.	

Pythagoras, for whom the famous theorem is named, lived during the 6th century B.C. on the island of Samos in the Aegean Sea, in Egypt, in Babylon and in southern Italy. Pythagoras was a teacher, a philosopher, a mystic and, to his followers, almost a god.

Pythagoras is often referred to as the first pure mathematician. Pythagoras was well educated, and he played the lyre throughout his lifetime, knew poetry and recited Homer. Pythagoras left Samos for Egypt in about 535 B.C. to study with the priests in the temples. In 520 BC, Pythagoras left Babylon and returned to Samos, and sometime later began a school called The Semicircle.

Pythagoras founded a philosophical and religious school where his many followers lived and worked. The Pythagoreans lived by rules of behavior, including when they spoke, what they wore and what they ate. Pythagoras was the Master of the society, and the followers were known as mathematikoi. They had no personal possessions and were vegetarians. Pythagoras believed:

All things are numbers. Mathematics is the basis for everything, and geometry is the highest form of mathematical studies. The physical world can be understood through mathematics.

Numbers have personalities, characteristics, strengths and weaknesses.

Some of the students of Pythagoras eventually wrote down the theories, teachings and discoveries of the group, but the Pythagoreans always gave credit to Pythagoras as the Master for:

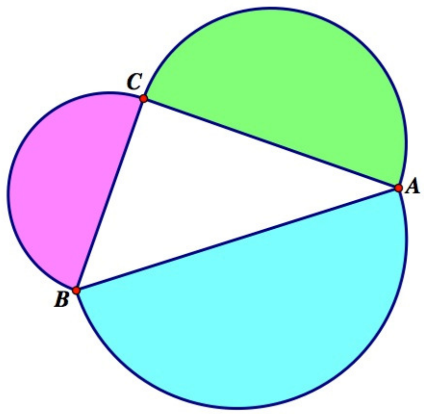
1. The sum of the angles of a triangle is equal to two right angles.
2. The theorem of Pythagoras - for a right-angled triangle the square on the hypotenuse is equal to the sum of the squares on the other two sides. Although the Babylonians understood this 1000 years earlier, Pythagoras proved it.
3. The discovery of irrational numbers is attributed to the Pythagoreans, but seems unlikely to have been the idea of Pythagoras because it does not align with his philosophy that all things are numbers, since number to him meant the ratio of two whole numbers.

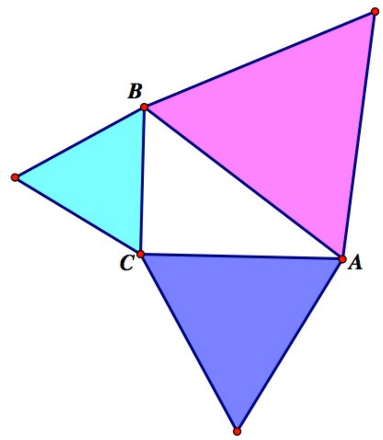
Pythagoras studied odd and even numbers, triangular numbers, and perfect numbers. Pythagoreans contributed to our understanding of angles, triangles, areas, proportion, polygons, and polyhedrons. Pythagoras also related music to mathematics. He had long played the seven string lyre, and learned how harmonious the vibrating strings sounded when the lengths of the strings were proportional to whole numbers, such as 2:1, 3:2, 4:3. Pythagoreans also realized that this knowledge could be applied to other musical instruments.

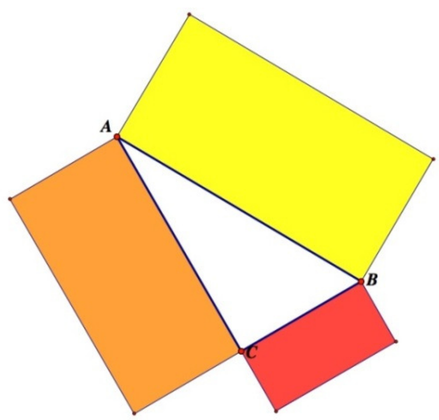
The Pythagorean Theorem is a cornerstone of mathematics, and continues to be so interesting to mathematicians that there are more than 400 different proofs of the theorem, including an original proof by President Garfield. The Pythagorean Theorem exhibits a fundamental truth about the way some pieces of the world fit together. Many mathematicians think that the Pythagorean Theorem is the most important result in all of elementary mathematics.

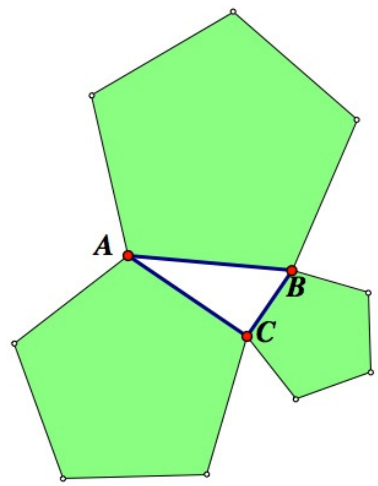
Activity 14-6: **Additional Pythagorean Theorem Trivia** Name:

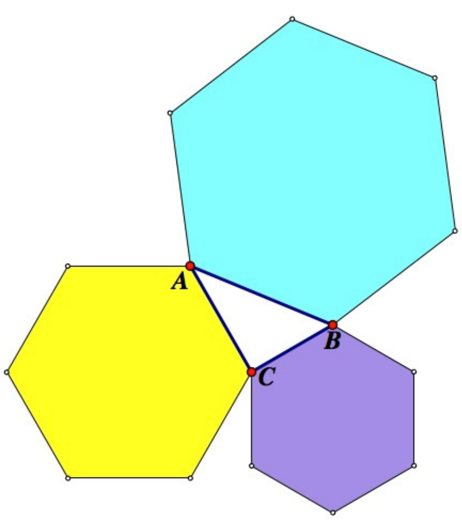
The Pythagorean Theorem does not just work for squares. It works for any similar shapes on the three sides of the triangle.

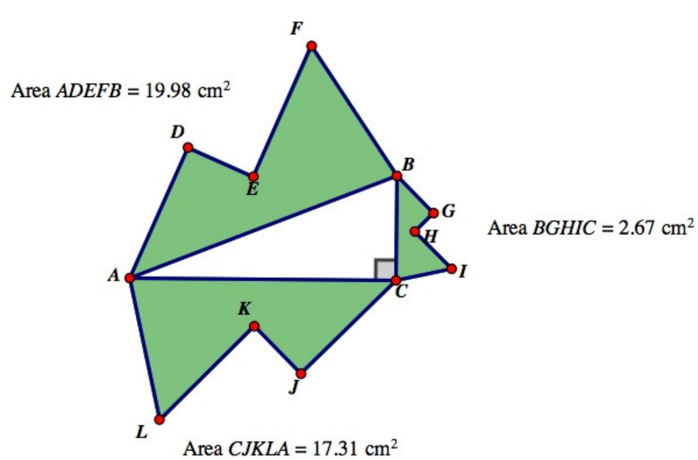








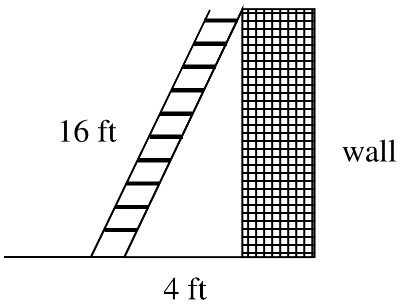




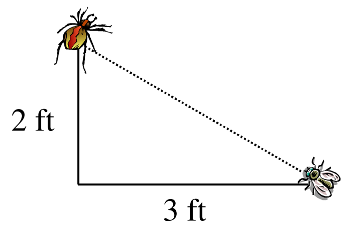
Activity 14-7: **Pythagorean Theorem** Name: ____________________

Write an equation that can be used to answer each question. Then solve. Simplify all radicals. Show all work on a separate sheet of paper.

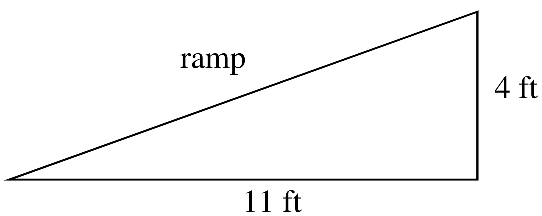
1. How high will the ladder reach?



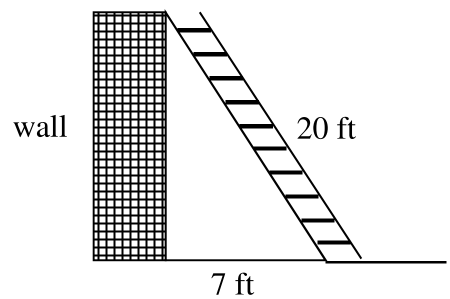
2. How far apart are the spider and the fly?



3. How long is the ramp?



4. How high will the ladder reach?



Pythagorean triples are three numbers that always work to solve a Pythagorean Theorem problem. For example, 3-4-5 is a Pythagorean triple. If you multiply all of the numbers by the same multiple, you will have another Pythagorean triple. Therefore, 30-40-50 and 300-400-500 are Pythagorean triples in the same family as 3-4-5.

For each Pythagorean triple, find two triples in the same family.

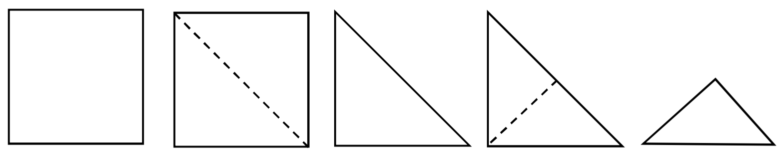
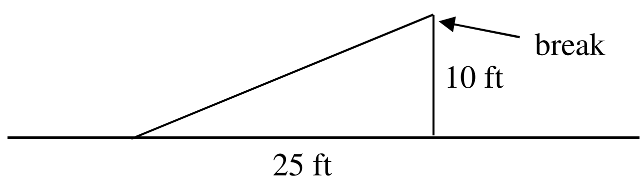
5.	7-24-25		6.	5-12-13	

Find the missing measurement. You may want to draw a picture to assist you.

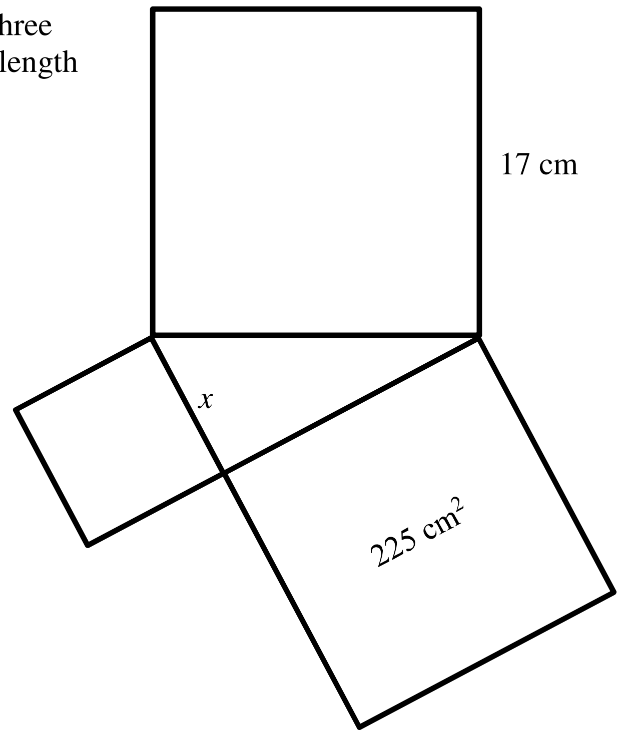
7.	How long is the diagonal of a rectangular table which is 5 ft wide and 18 ft long?	
8.	A helicopter flies 6 miles north and 9 miles east. How far is it from where it started?	
9.	The diagonal of a square is $8\sqrt{2}$. Find the length of each side of the square.	
10.	A kite is flying at the end of a 100 yard string. It is 40 yards above the ground. About how far away horizontally is the person holding the string from the kite?	

Activity 14-8: **Pythagorean Theorem** Name: ______________________

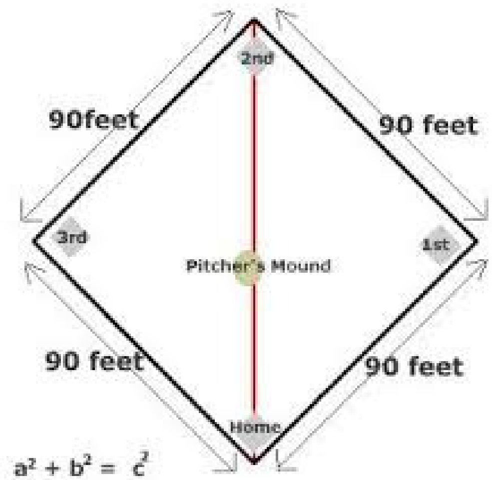
For each problem without a picture, first draw a picture of the problem. Then write an equation that can be used to answer each question. Then solve. Simplify all radicals unless the problem states something different. Show all work on a separate sheet of paper.

1.	A football field is 100 yards long from goal line to goal line and 53 yards wide from side to side. What is the longest possible straight line you could draw on the football field? (calculator)	
2.	Start with a 4 inch square. Fold it in half along the diagonal. Now fold it in half along the dotted line as shown. What is the perimeter of the final triangular-shaped paper? 	
3.	You are walking along and notice a telephone pole that has been broken. Ten feet of the pole is still standing and you also notice that the top of the pole is now 25 feet away from the base. How high was the telephone pole before it was broken? 	
4.	A utility pole 10 m high is supported by two guy wires. Each guy wire is anchored 3 m from the base of the pole. How many meters of wire are needed for the guy wires?	
5.	To meet federal guidelines, a wheelchair ramp that is constructed to rise 1 foot off the ground must extend 12 feet along the ground. How long will the ramp be? (calculator)	
6.	A park is 240 feet long and 180 feet wide. What is the length of a diagonal path that connects two corners of the park? (calculator)	
7.	Mr. Mangham drove 8 miles due east and then 5 miles due north. How far is Mr. Mangham from his starting point?	
8.	Chelsea is competing in a triathlon which is in the shape of a right triangle. The legs of the triangle are the 4 mile swim and the 10 mile run. How far is the biking part of the triathlon?	
9.	Mrs. Bailey, a circus performer, walks on a tightrope 25 feet above the ground. The tightrope is supported by two beams and two support cables. If the distance between each beam and the base of its support cable is 15 feet, what is the total length of all the support cable?	
10.	If the hypotenuse of triangle is 34 and one leg is 16, how long is the second leg?	

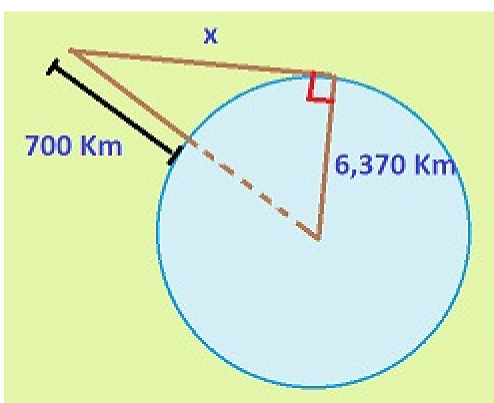
1. The right triangle shown below is formed by joining three squares at their vertices. What is the value of x, the side length of the smallest square?

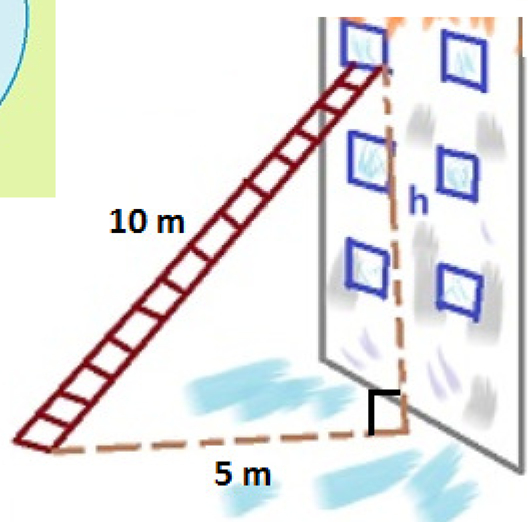


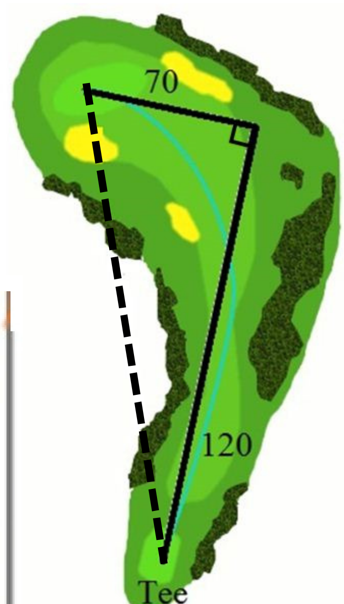
2. How far is a throw from home to 2[nd]?

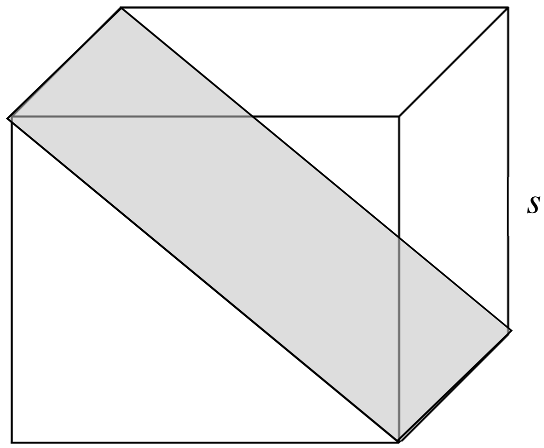


3-5. Find the missing lengths.









59% of Texas' 12th graders missed this TAKS problem.

Look at the **cube**. Which expression best represents the area of the shaded rectangle located diagonally in the cube?

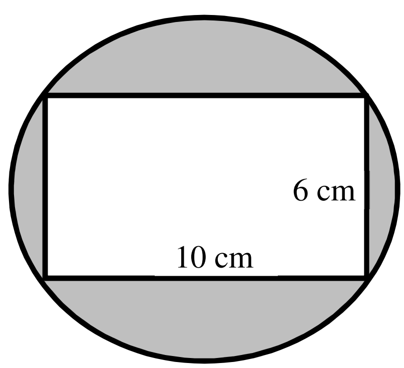
A. $A = s^2\sqrt{3}$

B. $A = \frac{s^3}{2}$

C. $A = s^3\sqrt{2}$

D. $A = s^2\sqrt{2}$

Determine the area of the shaded region.



The distance formula is a special version of the Pythagorean Theorem. To find the distance between two points, create a right triangle and then determine how long the hypotenuse would be. The actual distance formula is:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

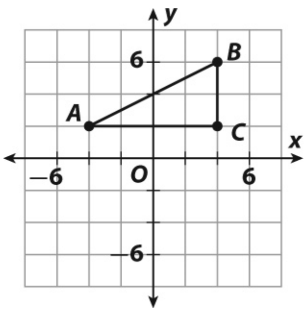
Using a piece of graph paper or paper with coordinate planes, place the following pair of points correctly on the plane. Then, using the Pythagorean Theorem, determine the distance between the points. Simplify all radicals. Show all work on a separate sheet of paper.

1.	(2, 3) and (5, 6)		2.	(-3, 2) and (3, 4)	
3.	(1, 3) and (3, -3)		4.	(-3, 2) and (2, -3)	
5.	(5, 4) and (5, -3)		6.	(1, 2) and (2, 4)	
7.	(-2, 0) and (1, 3)		8.	(-2, -3) and (3, -4)	
9.	(3, -1) and (5,6)		10.	(6,2) and (1,3)	
11.	(7,3) and (-2,-2)		12.	(-5, -5) and (3,-4)	

Find the missing measurement. Simplify all radicals. You may want to draw a picture to assist you.

13.	A carpenter braces an 8 ft by 15 ft wall by nailing a board diagonally across the wall. How long is the bracing board?	
14.	The lengths of the sides of a right triangle are given by three consecutive integers. Find the lengths of the sides.	
15.	A wire is stretched from the top of a 4 ft pole to the top of a 9 ft fence. If the pole and fence are 12 ft apart, how long is the wire?	
16.	Susan planted a Bradford pear tree 12 feet west and 1 foot north of her flagpole. She also planted a Juniper tree 15 feet east and 3 feet north of her flagpole. How far apart are the two trees?	
17.	Harry, Ron, and Hermione all attend the same school. Harry lives 11 mi west and 10 mi north of the school. Ron lives 13 mi east and 8 mi south of the school. Hermione lives 12 mi south and 9 mi west of the school. Who lives closest to the school and who lives farthest?	

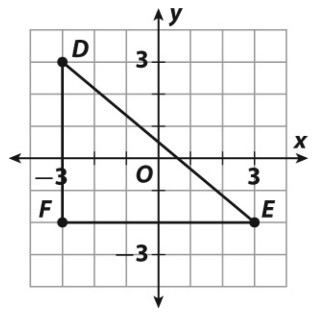
Name the coordinates of the points.

1. 

A(________ , ________)

B(________ , ________)

C(________ , ________)

2. 

D(________ , ________)

E(________ , ________)

F(________ , ________)

Name the hypotenuse of each right triangle in problems 1 and 2.

3. Hypotenuse in problem 1: ______________________________

4. Hypotenuse in problem 2: ______________________________

Estimate the length of the hypotenuse for each right triangle in problems 1 and 2.

5. Hypotenuse in problem 1: ______________________________

6. Hypotenuse in problem 2: ______________________________

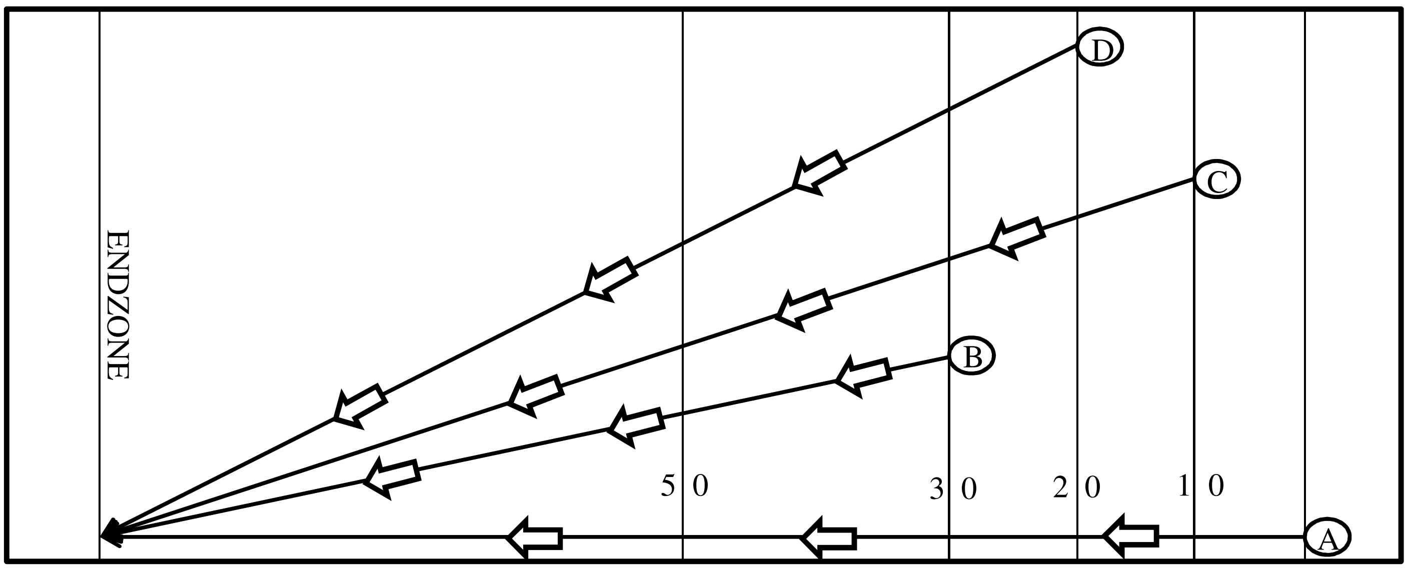
Use the distance formula to calculate the length of the hypotenuse for each right triangle.

7. Hypotenuse in problem 1: ______________________________ ______________________________

8. Hypotenuse in problem 2: ______________________________ ______________________________

9. Use the distance formula to find the distance between the points $(-4, -4)$ and (4, 4).

____



You may use a calculator for calculations. In questions 1-4, round all yards to the nearest tenth. In questions 5-8, round all seconds to the nearest tenth. In the bonus question, round all yards to the nearest yard.

Include all work on a separate sheet of paper.

Player A made an interception at the goal line and is trying to return the ball for a touchdown (100 yards). Players B, C, and D are all chasing him down and trying to tackle him before he gets there. **Important information:** Player B starts from the 30 yard line and 20 yards above Player A. Player C starts from the 10 yard line and 40 yards above Player A. Player D starts at the 20 yard line and 50 yards above Player A.

How long will it take to reach the endzone if…

1.	How far does Player A have to run to reach the endzone?		5.	Player A runs 10 yards/sec.	
2.	How far does Player B have to run to reach the endzone at the same point as Player A?		6.	Player B runs 7 yards/sec.	
3.	How far for Player C?		7.	Player C runs 9 yards/sec.	
4.	How far for Player D?		8.	Player D runs 9.5 yards/sec.	
9.	Will one of the players be able to tackle Player A before he reaches the endzone? If so, who? If more than one will, which player will get there first?				
BONUS	Let's say we wanted to have all four players reach the endzone in exactly 10 seconds. How many yards forward or backward would you need to place Player B to get to the endzone in exactly 10 seconds? Player C? Player D?				